2 Theoretical background of the methodologies and models implemented in R-CRISIS

2.1 Seismicity models

Generally speaking, R-CRISIS expects to have the seismicity described by means of the probabilities of having 1, 2, ..., *N* earthquakes of given magnitudes, at a given location, during the next T_f years. As can be noted by the reader, this is the most general description of seismicity that can possibly be given.

To get this information, R-CRISIS admits three different types of seismicity models. The first two are related to Poissonian occurrences, although they differ in the way in which the earthquake magnitude exceedance rates are defined, whereas the third model corresponds to a generalized non-Poissonian model where the required probabilities are explicitly provided by the user to the program. A complete description of each seismicity model implemented in R-CRISIS is provided next.

2.1.1 Modified Gutenberg-Richter model

This model is associated to Poissonian occurrences and so, the probability of exceeding the intensity level *a* in the next T_f years, given that an earthquake with magnitude *M* occurred at a distance *R* from the site of interest, is described by:

$$Pe(a,T|M,R)=1-\exp\left[-\Delta\lambda(M)T\cdot p_1(a|M,R)\right] \quad \text{Eq. (2-1)}$$

where $P_e(a|M,R)$ is the exceedance probability of the hazard intensity level *a*, given that an event with magnitude *M* occurred at a distance *R* from the site of interest, and $\Delta\lambda(M)$ is the Poissonian magnitude exceedance rate associated to the magnitude range (also denoted herein as magnitude bin) characterized by magnitude *M*. Note that $P_e(a|M,R)$ depends only on the magnitude and the site-to-hypocenter distance and therefore, this probability does not depend on earthquake occurrence probabilities.

On the other hand, $\Delta\lambda(M)$ can be computed as

$$\Delta\lambda(M)=\lambda\left(\frac{M-\Delta M}{2}\right)-\lambda\left(\frac{M+\Delta M}{2}\right) \quad \text{Eq. (2-2)}$$

where it is implicit that the magnitude bin characterized by magnitude *M* covers the range between M-$\Delta M/2$ and $M+\Delta M/2$. For the modified Gutenberg-Richter model (Cornell and Vanmarke, 1969), the earthquake magnitude exceedance rate is given by:

$$\lambda(M)=\lambda_0\frac{\exp(-\beta M)-\exp(-\beta M_U)}{\exp(-\beta M_0)-\exp(-\beta M_U)}, M_0\le M\le M_U \quad \text{Eq. (2-3)}$$

where λ_o is the exceedance rate of the threshold magnitude, M_o; β is a parameter equivalent to the "*b*-value" for the source (except that it is given in terms of its natural logarithm) and M_U is the maximum magnitude associated to the seismic source.

R-CRISIS can account for uncertainties in both β and M_U. On the one hand and to handle the uncertainty in the β parameter, the user must provide its expected value and its coefficient of variation (*CoV*); on the other hand, and in order to handle the uncertainty in the M_U value, its expected value and standard deviations are needed. More details about the treatment of those uncertainties are explained next.

Uncertainty in β value

Using a Bayesian framework, R-CRISIS treats λ_o and β parameters as independent random (and unknown) variables. Moreover, it assumes that uncertainty in β is correctly described by means of a Gamma probability distribution and, for the reasons described later, it disregards uncertainty in λ_o.

To explain the soundness of this treatment, the following commonly accepted hypotheses are assumed:

1. Occurrences are Poissonian
2. The probability distribution of magnitudes follows a Gutenberg-Richter (G-R) relation that is unbounded at the right-hand side. This is to say that the maximum possible magnitude, M_U, is much larger than M_o.

A consequence of the first assumption is that the times between earthquakes with magnitude $M \geq M_o$, τ, are independent, equally distributed random variables that follow an exponential distribution. Thus, its associated probability density function is:

$$p_T(\tau) = \lambda_0 e^{-\lambda_0 \tau} \qquad \text{Eq. (2-4)}$$

where λ_o is an unknown parameter. Also, it follows from hypothesis 1 that the times of earthquake occurrences, and their corresponding magnitudes, are independent from each other. From hypothesis 2 it is implied that magnitudes are independent too and are represented by means of equally distributed random variables with a shifted exponential distribution. Therefore, their probability density function is:

$$p_M(M) = \beta e^{-\beta(M-M_0)} \qquad \text{Eq. (2-5)}$$

where β is also an unknown parameter. It can be verified that equation 2-4 integrates to unity in the range of $\tau \geq 0$ while equation 2-5 integrates to 1.0 in the range $M \geq M_o$ (remember that, until now, M is unbounded).

Now, consider the observation of an event consisting in the occurrence of N earthquakes, with inter-event times, τ_i, and magnitudes M_i, i=1..N. According to the assumptions mentioned before, the likelihood of this event, given unknown parameters θ=(λ_0, β) can be written as:

$$l(\varepsilon|\theta) = \prod_{i=1}^{N} \lambda_0 e^{-\lambda_0 \tau_i} \beta e^{-\beta(M_i - M_0)}$$ Eq. (2-6)

Or, in other words,

$$l(\varepsilon|\theta) = \lambda_0^N e^{-\lambda_0 \sum_i \tau_i} \beta^N e^{-\sum_i \beta(M_i - M_0)}$$ Eq. (2-7)

From equation 2-7, the classic maximum likelihood estimators for λ_0 and β can be estimated:

$$\widehat{\lambda_0} = \frac{N}{\sum_i \tau_i} = \frac{N}{T}$$ Eq. (2-8)

$$\widehat{\beta} = \frac{N}{\sum_i \beta(M_i - M_0)}$$ Eq. (2-9)

where $T = \sum_i \tau_i$ is the total observation time in the catalog for the selected threshold magnitude, M_0.

Continuing with the use of a Bayesian approach, λ_0 and β are regarded as random variables whose probability distributions are fixed *a priori* and then updated in the light of the earthquake observations (Newmark and Rosenblueth, 1971).

A common approach is to use as prior distributions the natural conjugates of the process. In this case, an examination of the likelihood function in equation 2-7 shows that the following likelihood (the kernel of the probability function) is the natural conjugate of the process:

$$l(\theta) = \lambda_0^{r-1} e^{-u\lambda_0} \beta^{k-1} e^{-s\beta}$$ Eq. (2-10)

where, under the *a priori* Bayesian approach, the expected value of β is k/s and its CoV is equal to $1/\sqrt{s}$. On the other hand, the expected value of λ_0 is r/u and its CoV equal to $1/\sqrt{r}$.

The selected prior is the product of two Gamma distributions. Then, applying Bayes' theorem, the posterior distribution of the unknown parameters is found.

$$l(\theta|\varepsilon) = l(\varepsilon|\theta) l(\theta) = \lambda_0^{N+r-1} e^{-\lambda_0 (u + \sum_i \tau_i)} \beta^{N+k-1} e^{-\beta(s + \sum_i (M_i - M_0))}$$ Eq. (2-11)

It is evident that, *a posteriori*, both λ_o and β are Gamma distributed but, more relevant for this explanation, it can be observed that, *a posteriori*, they are independent from each other since the joint posterior likelihood of θ is simply the product of the likelihoods of λ_o and β.

The result is perhaps unexpected for those not familiar with the use of Bayesian methods (now the user can see that the maximum likelihood approach is a particular case of the more general Bayesian method), but it is intuitively correct. It is correct to say that one is estimating λ_o and β with the maximum likelihood method (equations 2-8 and 2-9). Now say that after a first estimation round, one discovers that one of the magnitudes in the sample was wrong. This new information, as can be seen from equations 2-8 and 2-9, would change the estimation of β, but it would not change the estimation of λ_o, which is basically a rate.

Equation 2-11 justifies two important features of R-CRISIS:

1. Treating λ_o and β as independent (provided, of course, that they have been estimated by Bayesian methods or, at least, with the maximum likelihood method);
2. Treating the uncertainty in β assuming that this variable follows a Gamma distribution.

Equation 2-11, by the way, also provides information about the size of the uncertainty in β: *a posteriori*, since its *CoV* is:

$$CoV(\beta) = \frac{1}{\sqrt{(N+k-1)}} \qquad \text{Eq. (2-12)}$$

so, if the prior information is not very large (that is, if $r << N$, meaning that the sample size is reasonably large) then its coefficient of variation is of the order of $1/N^{1/2}$.

Now, we will remove the restriction that $M_U >> M_o$. R-CRISIS estimates the magnitude exceedance rate following a modified G-R relationship, provided by equation 2-3 and for this case, the probability density function of M is the following:

$$p_M(M) = \beta \frac{e^{-\beta(M-M_0)}}{1-e^{-\beta(M_U-M_0)}} \qquad \text{Eq. (2-13)}$$

Replacing equation 2-13 into equation 2-7 and considering that nothing has changed related to the occurrence times, it can be found that:

$$l(\varepsilon \mid \theta) = \lambda_0^N e^{-\lambda_0 \sum_i \tau_i} \beta^N \frac{e^{-\sum_i \beta(M_i-M_0)}}{(1-e^{-\sum_i \beta(M_U-M_0)})^N} \qquad \text{Eq. (2-14)}$$

Now, the maximum likelihood estimators cannot be determined analytically (although, in general, they do not differ by much from those obtained with equations 2-8 and 2-9). But, if we continue with the Bayesian process, we can find that, although β is not Gamma distributed anymore (although its distribution is not far from a Gamma if M_O and M_U are not close enough), λ_O and β remain independent, *a posteriori*, due to the fact that λ_O is not present in the β-related term of the event likelihood. Because of this, the posterior joint likelihood of θ is again, simply the product of the likelihoods of λ_O and β.

The reason why R-CRISIS disregards uncertainty in λ_O is the following: consider that the basic seismic hazard equation, expressed in terms of intensity exceedance rates (even if a similar analysis could be performed for exceedance probabilities in given time frames), for a single point-source located at distance R from the site of analysis is:

$$\upsilon(a|\lambda_0,\beta)=\lambda_0\int_{M_0}^{M_U} p_M(M)\cdot \Pr(A>a|M,R)dM \qquad \text{Eq. (2-15)}$$

where $\nu(a|\lambda_O,\beta)$ is the exceedance rate of the hazard intensity a given that λ_O and β are known. Replacing equation 2-13 into equation 2-15 we find that:

$$\upsilon(a|\lambda_0,\beta)=\lambda_0\int_{M_0}^{M_U} \beta\frac{e^{-\beta(M-M_0)}}{1-e^{-\beta(M_U-M_0)}}\cdot \Pr(A>a|M,R)dM \qquad \text{Eq. (2-16)}$$

To remove the conditionality in $\nu(a)$ we integrate with respect to the joint probability density function of the unknown parameters (λ_O and β in this case), which amounts to computing its expected value with respect to them:

$$\upsilon(a)=\int\int \upsilon(a|\lambda_0,\beta)p_{\beta,\lambda_0}(\beta,\lambda_0)d\beta d\lambda_0 \qquad \text{Eq. (2-17)}$$

Since it was already established that λ_O and β are independent random variables, it can be said that:

$$\upsilon(a)=\int\int \upsilon(a|\lambda_0,\beta)p_{\beta}(\beta)p_{\lambda_0}(\lambda_0)d\beta d\lambda_0 \qquad \text{Eq. (2-18)}$$

and, since the distribution of β does not depend on λ_O, $\nu(a)$ is:

$$\upsilon(a)=\int \lambda_0 p_{\lambda_0}(\lambda_0)d\lambda_0\int\int_{M_0}^{M_U} \frac{e^{-\beta(M-M_0)}}{1-e^{-\beta(M_U-M_0)}}\Pr(A>a|M,R)p_{\beta}(\beta)dMd\beta \qquad \text{Eq. (2-19)}$$

Therefore,

$$\upsilon(a) = E(\lambda_0) E_\beta \left\{ \int_{M_0}^{M_U} \frac{e^{-\beta(M-M_0)}}{1-e^{-\beta(M_U-M_0)}} \Pr(A > a \mid M,R) dM \right\} \quad \text{Eq. (2-20)}$$

where $E_\beta(\cdot)$ denotes the expected value with respect to β. It is clear from equation 2-20 that the first probability moment of the exceedance rate (the quantity usually reported as "the" exceedance rate) is insensitive to uncertainty in λ_o but, since $\nu(a)$ depends on the probability distribution assigned to β (we need this distribution to compute the expected value with respect to β), it definitively depends on the uncertainty of β.

In summary, to compute the expected value of the exceedance rates, R-CRISIS solves equation 2-20 for point-sources, generated from the subdivision of the sources originally given by the user (see Section 2.2.1), using a Gamma distribution to describe the uncertainty in β. Since exceedance rates are additive, so are their expected values. Hence, disregarding uncertainty in λ_o for computing the first probability moment of the intensity exceedance rate is rigorously justified.

Note from equation 2-20 that disregarding uncertainty in β would be equivalent to replacing the probability density function assigned to this parameter with the following Dirac's delta function:

$$p_\beta(\beta) = \delta\left[\beta - E(\beta)\right] \quad \text{Eq. (2-21)}$$

In that case, equation 2-20 would take the following form:

$$\upsilon(a) = E(\lambda_0) \int_{M_0}^{M_U} \frac{e^{-E(\beta)(M-M_0)}}{1-e^{-E(\beta)(M_U-M_0)}} \Pr(A > a \mid M,R) dM \quad \text{Eq. (2-22)}$$

which is evidently, the classic seismic hazard equation (compare against equation 2-16) when parameters λ_o and β are deterministically equal to their respective expected values. In general, however, equation 2-20 must be considered only a first-order approximation to the true value of the seismic hazard intensity exceedance rate.

Clearly, if higher-order moments of $\nu(a)$ are required, a correct answer could only be obtained by accounting for the uncertainty in λ_o. Anyhow, since R-CRISIS reports only the expected value of the intensity exceedance rates, there is no need to know how uncertain λ_o is.

Note: From R-CRISIS v20, the same methodology to consider uncertainty of the β-value has been maintained but its incorporation into synthetic catalogues has been optimized.

Uncertainty in the maximum magnitude

R-CRISIS regards the maximum magnitude, M_U, as an unknown quantity. It is possible to assign to this variable a uniform probability distribution between M_{U1} and M_{U2} (see Figure 2-

1), which are informed to R-CRISIS in terms of two values: the expected value of M_U, $E(M_U)$, and σM. If $\sigma M<0.5$, M_U is treated in a deterministic way with a weight concentration equal to 1.0 at $M_U=E(M_U)$. But, if $\sigma M\geq 0.5$, R-CRISIS generates five probability concentrations centered at $E(M_U)$ with a uniform density between M_{U1} and M_{U2} that correspond to the values indicated by equations 2-23 and 2-24.

$$M_{U1} = E(M_U) - \sigma M \qquad \text{Eq. (2-23)}$$

$$M_{U2} = E(M_U) + \sigma M \qquad \text{Eq. (2-24)}$$

Thus, maximum magnitude is considered equally likely for all values between M_{U1} and M_{U2}.

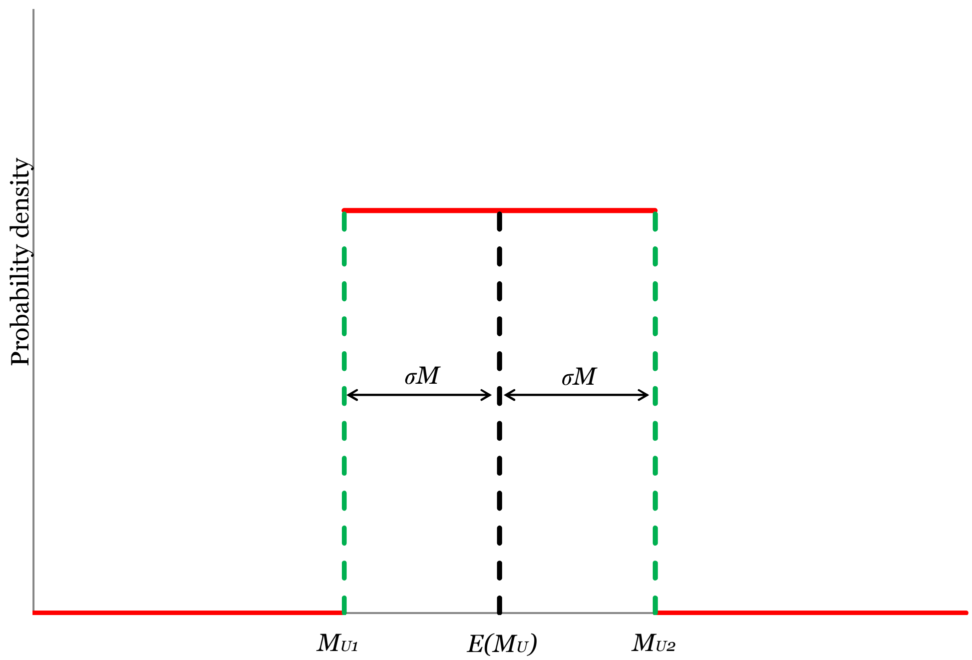

Figure 2-1 Probability density function of the M_U value

2.1.2 Characteristic earthquake model

This seismicity model is also associated to Poissonian occurrences and therefore, the probability of exceeding the intensity level, *a*, in the next T_f years, given that an earthquake with magnitude *M* occurred at a distance *R* from the site, is again given by equation 2-1 with the same considerations and assumptions explained before.

For the Characteristic Earthquake model (Youngs and Coppersmith, 1985) implemented in R-CRISIS, the earthquake magnitude exceedance rate is given by:

$$\lambda(M) = \lambda_o \frac{\Phi\left[\frac{M_U - EM}{s}\right] - \Phi\left[\frac{M - EM}{s}\right]}{\Phi\left[\frac{M_U - EM}{s}\right] - \Phi\left[\frac{M_o - EM}{s}\right]}, M_o \le M \le M_U \qquad \text{Eq. (2-25)}$$

where $\Phi[\cdot]$ is the standard normal cumulative function and M_o and M_U are the threshold and maximum characteristic magnitudes, respectively; *EM* and *s* are, on the other hand, parameters that define the distribution of *M*.

EM can be interpreted as the expected value of the characteristic earthquake and *s* as its standard deviation. λ_o is the exceedance rate of magnitude M_o. In addition, a slip-predictable behavior can be modeled assuming that *EM* grows with the time elapsed since the last characteristic event, *T*00, in the following way:

$$E(M) = D + F\ln(T00) \qquad \text{Eq. (2-26)}$$

Note: if *F* is set to zero, then *EM* is equal to *D*, independently of the time elapsed.

2.1.3 Generalized non-Poissonian model

This type of seismicity description allows specifying directly the required probabilities, that is, the probabilities of having 1, 2, ..., *Ns* earthquakes of given magnitudes, at a given location, during the next T_f years.

This information is provided by the user to R-CRISIS by means of a binary file, with *.nps[5] extension, which has the structure explained in Tables 2-1 and 2-2.

[5] Non-Poissonian Seismicity

Table 2-1 Generalized seismicity file structure (part 1)

Generalized seismicity file				
Description	**Variable**	**Type**	**Length**	**Comments**
Number of point sources	TotSrc	Integer	4	-
Number of magnitude bins	Nbin	Integer	4	-
Number of time frames	Nt	Integer	4	-
Maximum number of events for which Prob(i,j) is given	Ns	Integer	4	-
Magnitude representative of bin 1	M(1)	Double	8	Magnitude values are useful only if parametric attenuation models are used. They are not used in generalized attenuation models
...	...	...	...	
Magnitude representative of bin Nbin	M(Nbin)	Double	8	
Time frame 1	Tf(1)	Double	8	-
...	...	...	...	-
Time frame Nt	Tf(Nt)	Double	8	-
Seismicity record for source 1	Seis(1)	Seismicity record	8+8*Ns*Nt*Nbin	-
Seismicity record for source 2	Seis(2)	Seismicity record	8+8*Ns*Nt*Nbin	-
...	...	...	...	-
Seismicity record for source TotSrc	Seis(TotSrc)	Seismicity record	8+8*Ns*Nt*Nbin	-

Table 2-2 Generalized seismicity file structure (part 2)

Seismicity record				
	Variable	**Type**	**Length**	**Description**
Probability of having 1, 2,…,Ns events of magnitude 1 in time frame 1	Prob(1,1,1)	Double	8	Block associated to Magnitude 1
	Prob(1,1,2)	Double	8	
	…	-	-	
	Prob(1,1,Ns)	Double	8	
Probability of having 1, 2,…,Ns events of magnitude 1 in time frame 2	Prob(1,2,1)	Double	8	
	Prob(1,2,2)	Double	8	
	…	-	-	
	Prob(1,2,Ns)	Double	8	
…	…	…	…	
Probability of having 1, 2,…,Ns events of magnitude 1 in time frame Nt	Prob(1,Nt,1)	Double	8	
	Prob(1,Nt,2)	Double	8	
	…	-	-	
	Prob(1,Nt,Ns)	Double	8	
…	…	…	…	…
Probability of having 1, 2,…,Ns events of magnitude Nbin in time frame 1	Prob(Nbin,1,1)	Double	8	Block associated to Magnitude Nbin
	Prob(Nbin,1,2)	Double	8	
	…	…	…	
	Prob(Nbin,1,Ns)	Double	8	
Probability of having 1, 2,…,Ns events of magnitude Nbin in time frame 2	Prob(Nbin,2,1)	Double	8	
	Prob(Nbin,2,2)	Double	8	
	…	…	…	
	Prob(Nbin,2,Ns)	Double	8	
	…	…	…	
Probability of having 1, 2,…,Ns events of magnitude Nbin in time frame Nt	Prob(Nbin,Nt,1)	Double	8	
	Prob(Nbin,Nt,2)	Double	8	
	…	…	…	
	Prob(Nbin,Nt,Ns)	Double	8	

2.1.4 Generalized Poissonian model

In this option, included in R-CRISIS by suggestion of Dr. Ramón Secanell, seismicity is described by means of a non-parametric characterization of the activity (or occurrence) rates of earthquakes of given magnitudes at one or several seismic sources.

Seismicity information is provided by the user to R-CRISIS in a text file, with *.gps[6] extension, which has the structure shown in Table 2-3

[6] Generalized Poissonian Seismicity

Table 2-3 Generalized Poissonian seismicity file structure

Description	Comments
ID Header	A line of text used for identification purposes
NumSources	Number of different sources whose seismicity is described in the file
NumBins	Number of magnitude bins in which the seismicity curve is discretized
Magnitude 1	Central point of magnitude bin 1
Magnitude 2	Central point of magnitude bin 2
....	...
Magnitude NumBins	Central point of magnitude bin NumBins
Δλ(1,1)	Occurrence rate of earthquakes with magnitude 1 in source 1
Δλ(2,1)	Occurrence rate of earthquakes with magnitude 2 in source 1
...	...
Δλ(NumBins,1)	Occurrence rate of earthquakes with magnitude NumBins in source 1
Δλ(1,2)	Occurrence rate of earthquakes with magnitude 1 in source 2
....	...
Δλ(NumBins,2)	Occurrence rate of earthquakes with magnitude NumBins in source 2
...	...
Δλ(NumBins,NumSources)	Occurrence rate of earthquakes with magnitude NumBins in source NumSources

The format of the *.gps file allows for the use of ":" as a separator (i.e. everything written before the separator is ignored by R-CRISIS). Table 2-4 shows an example of a *.gps file, describing the seismicity of four sources using 9 magnitude bins (please recall that everything written before ":" is ignored by R-CRISIS):

Table 2-4 Generalized Poissonian seismicity file example

Example of *.gps file
Four ModifiedGR sources with M0=4, Mu=8, Beta=1, Lambda0=1
NumSources: 4
NumBins: 9
Magnitude 1: 4.2222
Magnitude 2: 4.6667
Magnitude 3: 5.1111
Magnitude 4: 5.5556
Magnitude 5: 6.0000
Magnitude 6: 6.4444
Magnitude 7: 6.8889
Magnitude 8: 7.3333
Magnitude 9: 7.7778
Source 1 M=4.222222 : 0.5891
Source 1 M=4.666667 : 0.2422
Source 1 M=5.111111 : 0.0996
Source 1 M=5.555555 : 0.0409
Source 1 M=6.000000 : 0.0168
Source 1 M=6.444444 : 0.0069
Source 1 M=6.888888 : 0.0028
Source 1 M=7.333333 : 0.0012
Source 1 M=7.777777 : 0.0004
Source 2 M=4.222222 : 0.5891
Source 2 M=4.666667 : 0.2422
Source 2 M=5.111111 : 0.0996
Source 2 M=5.555555 : 0.0409
Source 2 M=6.000000 : 0.0168
Source 2 M=6.444444 : 0.0069
Source 2 M=6.888888 : 0.0028
Source 2 M=7.333333 : 0.0012
Source 2 M=7.777777 : 0.0004
Source 3 M=4.222222 : 0.5891
Source 3 M=4.666667 : 0.2422
Source 3 M=5.111111 : 0.0996
Source 3 M=5.555555 : 0.0409
Source 3 M=6.000000 : 0.0168
Source 3 M=6.444444 : 0.0069
Source 3 M=6.888888 : 0.0028
Source 3 M=7.333333 : 0.0012
Source 3 M=7.777777 : 0.0004
Source 4 M=4.222222 : 0.5891
Source 4 M=4.666667 : 0.2422
Source 4 M=5.111111 : 0.0996
Source 4 M=5.555555 : 0.0409
Source 4 M=6.000000 : 0.0168
Source 4 M=6.444444 : 0.0069
Source 4 M=6.888888 : 0.0028
Source 4 M=7.333333 : 0.0012
Source 4 M=7.777777 : 0.0004

Note that the values provided by this file are the occurrence rates of earthquakes with magnitudes contained within a magnitude bin. In other words, R-CRISIS expects, for a magnitude bin between M_1 and M_2, with $M_2>M_1$, the number of earthquakes, per unit time, that this source generates with magnitudes between M_1 and M_2. For instance, if these occurrence rates were to be computed from a usual exceedance rate plot, $\lambda(M)$, the occurrence rate of earthquakes in the mentioned magnitude bin corresponds to $\lambda(M_1)$-$\lambda(M_2)$.

For seismic hazard computation purposes, earthquakes generated in this source will have only the magnitudes given in the file as the central points of the various bins. Therefore, it is the responsibility of the user to give a magnitude discretization that is dense enough (which is a parameter that is user-defined in R-CRISIS).

This option was originally created specifically to be applied with the smoothed seismicity method developed by Woo (1996). Therefore, this option is frequently used to describe the seismicity of numerous point sources whose geometrical properties (e.g., location, rupture planes) are given by means of an *ssg[7] file (see Section 2.2.5). In this case, R-CRISIS interprets that each source described in this seismicity file corresponds to a point source described in the *.ssg file.

However, this Generalized Poisson model can be used to describe, in a non-parametric manner, the seismicity of area and/or line sources. For these cases, R-CRISIS will interpret that the occurrence rates provided in the *.gps file are associated to the whole source (area or line), and then, R-CRISIS will uniformly distribute the occurrence rate across or along it, depending if the geometry is described by means of an area or a line.

2.2 Geometry models

R-CRISIS has implemented different geometry models to describe the characteristics of the seismic sources. The available geometry models in R-CRISIS are:

a) Area sources (where area planes and volumes correspond to particular cases) that are modelled as planes by means of a set of vertexes that account for a three-dimensional representation.
b) Line sources that are modeled as polylines with constant or variable depths.
c) Point sources (where grid sources are a particular case).

The following sections provide a complete description of the geometry models implemented in R-CRISIS together with an explanation about how they are treated within the PSHA framework.

Note: within the same seismic hazard project, R-CRISIS allows the combination of different geometry models for different sources.

[7] SSG stands for: smoothed seismicity geometry

2.2.1 Area sources

When this geometry model is chosen, the seismic sources are modelled as polygons defined by the 3D coordinates for each of their vertexes. Figure 2-2 shows an example consisting of a 3D polygon with 8 vertexes representing a dipping plate, which also has a varying dip angle.

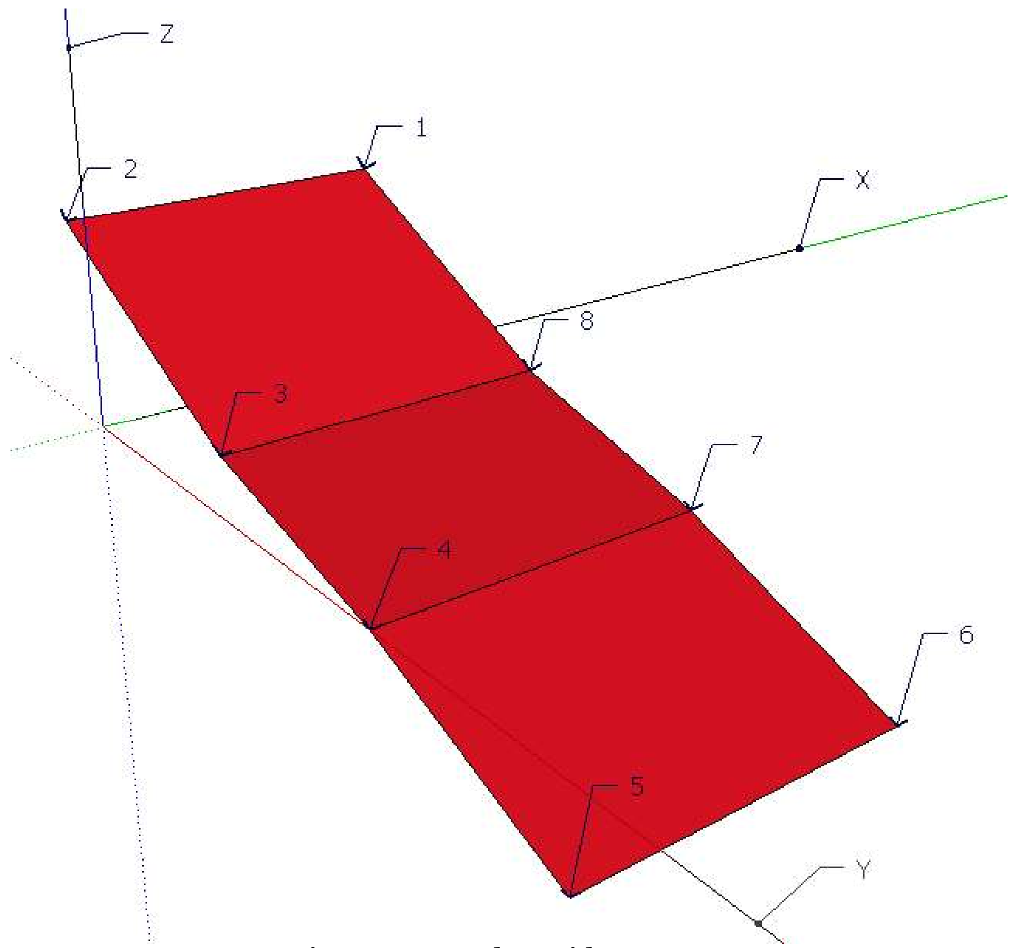

Figure 2-2 Area plane with 8 vertexes

Note: vertical planes are allowed in R-CRISIS.

In the case of area sources, and to perform the spatial integration (see Section 2.6), R-CRISIS divides the polygon into triangles using the routine explained with detail in Annex 1. In summary, R-CRISIS first checks if the triangulation can be made in the XY plane as shown in Figure 2-3 in terms of six triangles of different colors.

Note: the numbering of the vertexes of the area source must be provided in counter-clockwise order when this plane is seen from above the Earth's surface.

In the cases of vertical planes, R-CRISIS will try to triangulate the area in the XZ plane, so for these cases, the numbering of the vertexes must be done counter-clockwise in said plane.

Finally, R-CRISIS will try to perform the triangulation in the YZ plane. It is important to bear in mind that there are some particularly complicated source geometries that cannot be well triangulated by R-CRISIS (e.g.an L-shaped vertical plane) and then for these cases, an error will be reported.

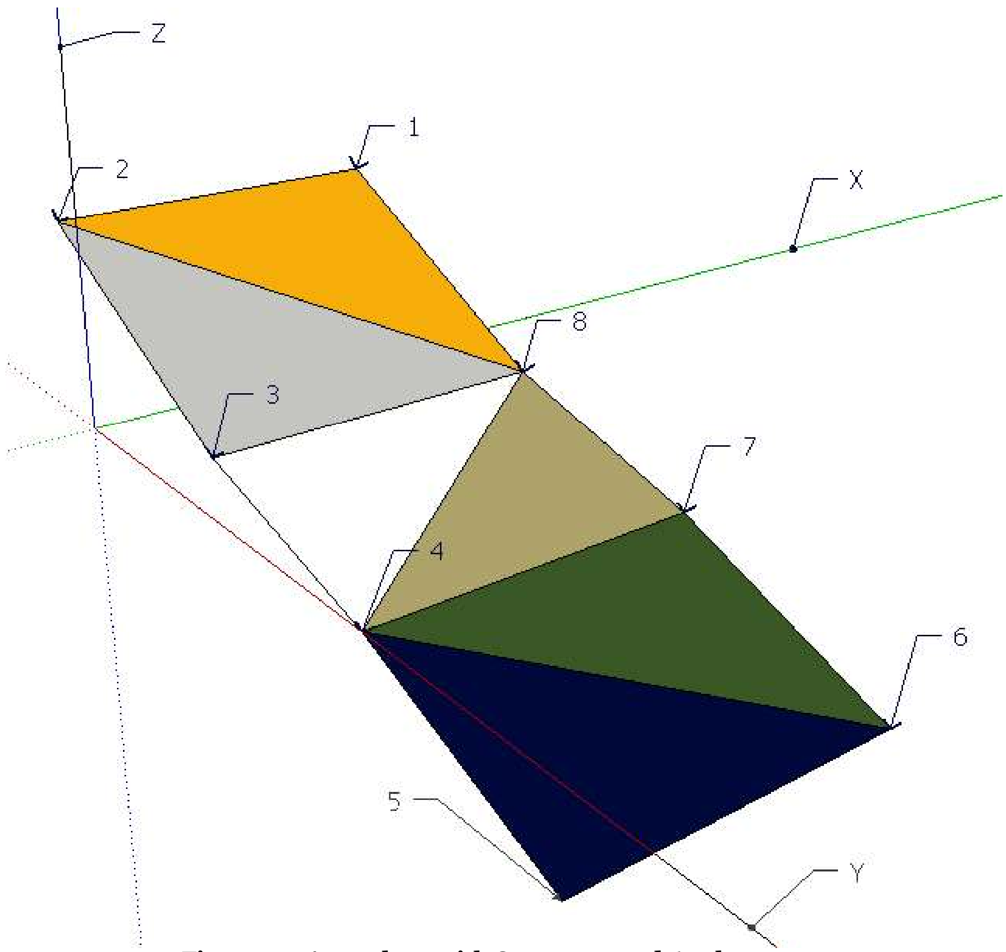

Figure 2-3 Area plane with 8 vertexes and 6 sub-sources

Note: to guarantee a good triangulation process, vertexes used to define the same seismic source cannot be closer than the perimeter of the source/1000000 (in m).

Relation between magnitude and rupture area

In R-CRISIS, attenuation relations (or ground motion prediction equations GMPE) can be specified in terms of 4 different distance measures (see Section 2.3). If R_{RUP} or R_{JB} distances are used, R-CRISIS requires means to know the rupture area (or length), as a function of magnitude, to compute the appropriate values for the distances.

For area and smoothed seismicity (gridded) sources, R-CRISIS initially assumes a circular rupture which radius *R* (in km) relates with the magnitude *M* in the following manner:

$$A = \pi R^2 \qquad \text{Eq. (2-27)}$$

where:

$$R = K_1 \cdot e^{K_2 M} \qquad \text{Eq. (2-28)}$$

and K_1 and K_2 are constants of the relationship between the magnitude and the rupture area.

Equation 2-27 can be rewritten thus as:

$$A = \pi K_1^{\,2} \cdot e^{2K_2 M} \qquad \text{Eq. (2-29)}$$

Several regression analyses performed to study the relationship between magnitude and rupture area (i.e. Wells and Coppersmith, 1994) adopt the following regression form:

$$\log A = a + bM \qquad \text{Eq. (2-30)}$$

where *A* is the rupture area, *M* is the magnitude and *a* and *b* are the regression coefficients. If Eq. 2-30 is rewritten as:

$$A = 10^a \cdot 10^{bM} \qquad \text{Eq. (2-31)}$$

equations 2-29 and 2-31 end with a similar structure with the following equivalences:

$$\pi K_1^{\,2} = 10^a \qquad \text{Eq. (2-32)}$$

$$e^{2K_2} = 10^b \qquad \text{Eq. (2-33)}$$

To verify the correctness of the equivalences shown in equations 2-32 and 2-33, in Tables 2-5 to 2-7 the regression coefficients, the R-CRISIS coefficients and the equivalences are shown.

Table 2-5 Wells and Coppersmith (1994) rupture area regression coefficients

Model	*a*	*b*
Strike-slip	-3.42	0.90
Reverse	-3.99	0.98
Normal	-2.87	0.82
All	-3.49	0.91

Table 2-6 R-CRISIS rupture area coefficients for the Wells and Coppersmith (1994) model

Model	*K1*	*K2*
Strike-slip	0.01100	1.03616
Reverse	0.00571	1.12827
Normal	0.02072	0.94406
All	0.01015	1.04768

Table 2-7 Equivalences between R-CRISIS and Wells and Coppersmith (1994) rupture area coefficients

Model	Eq 2-32		Eq 2-33	
	10^a	πK_1^2	10^b	e^{2K_2}
Strike-slip	3.80E-04	3.80E-04	7.943	7.943
Reverse	1.02E-04	1.02E-04	9.550	9.550
Normal	1.35E-03	1.35E-03	6.607	6.607
All	3.24E-04	3.24E-04	8.128	8.128

R-CRISIS has the built-in sets of constants, proposed by well-known authors (Brune, 1970; Singh et al., 1980; Wells and Coppersmith, 1994), as shown in Table 2-8.

Table 2-8 Built-in *K1* and *K2* constants

Model	*K1*	*K2*
Brune (1970)	0.00381	1.15130
Singh et al. (1980)	0.00564	1.15300
Wells and Coppersmith (1994) - Strike-slip	0.01100	1.03616
Wells and Coppersmith (1994) - Reverse	0.00571	1.12827
Wells and Coppersmith (1994) - Normal	0.02072	0.94406
Wells and Coppersmith (1994) - All	0.01015	1.04768

As shown in Figure 2-4 and considering that at each location earthquakes with different magnitudes are likely to occur, depending on the magnitude the area rupture will change. Each circle in Figure 2-4 corresponds to the area rupture associated to earthquakes, occurring at the same location but with different *M*.

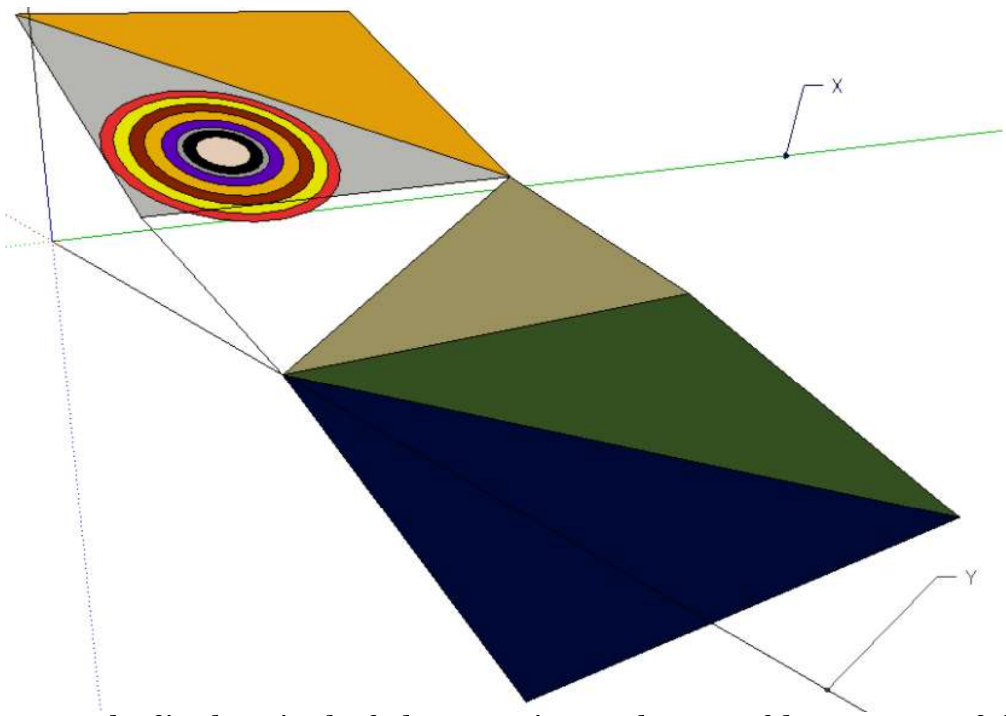

Figure 2-4 Example of in-plane circular fault ruptures in one sub-source of the area source of Figure 2-2

Orientation of the rupture plane

The orientation of the ruptures of the area sources are assigned by means of the values provided to R-CRISIS by the user in the strike field of the GUI. That value is to be provided in degrees. By default, R-CRISIS estimates an initial strike using the angle between vertexes 1 and 2 but this value can be changed by the user at any time.

Behavior options

R-CRISIS implements different models in which the rupture areas are modelled with differences ranging from the aspect ratio to the extent in which the fault can break. The different available options are explained with detail herein.

Normal

This is the default behavior in R-CRISIS for area sources. In general, the rupture areas are circular (i.e. ellipses with aspect ratio equal to 1.0), whose area is related to magnitude through parameters $K1$ and $K2$ as described in equation 2-27. For these cases, the rupture areas are contained in the plane of the source area itself and then, if the source area is a horizontal plane (that is, all its vertexes have the same depth) then the rupture planes will be horizontal whereas if the area source is a vertical plane, then the circles that represent the ruptures will be contained in a vertical plane. If the area geometry is complex (that is, it is a non-planar area), then the rupture plane will be that of the triangle in which the corresponding hypocenter is contained (see Figure 2-4). When this option is selected, it is important to bear in mind that R-CRISIS allows the rupture area to expand outside of the source area geometry (*leaky boundary*). If this behavior is not considered correct for the modelling purposes, then the behavior option **"treat as fault"** is suggested to be selected.

Treat as fault

The difference between area sources with normal or treat as fault behavior is that, for the latter case, R-CRISIS does not allow rupture areas to extend outside the limits defined by the geometry of the source (*strict boundary*). This difference is relevant only in the cases in which R_{RUP} or R_{JB} are used as distance measures and rupture areas are larger than 0 (i.e. parameters *K1* and *K2*>0).

To be possible in R-CRISIS than an area source is assigned the treat as fault behavior the following conditions must be met:

1. It must have 4 vertexes.
2. All vertexes must roughly be in the same plane (there are tolerances).
3. All internal angles of the polygon must be roughly straight (there are tolerances).

The tolerances for the verification about the vertexes being in the same plane is done by calculating a unit vector of vertex 1 by generating a triangle whose vertexes correspond to number 1, 2 and 4 of Figure 2-5 and then repeating the same calculation now for vertex 3 now generating a triangular plane by using vertexes 2, 3 and 4. The angle is estimated between the two normal vectors and if its difference is smaller than 1.146º, the source is considered as acceptable for the use of this behavior option.

The tolerances for the verification process about straight internal angles are the following; R-CRISIS calculates the values of the four internal angles using the geometry data provided by the user. If all the four internal angles are between 84.26º and 95.74º, the source is considered as acceptable for the use of this behavior option.

In this case the rupture areas will be elliptical with aspect ratio equal to the value provided to R-CRISIS by the user with an area related to magnitude through parameters *K1* and *K2*. The aspect ratio, *Ar* is defined as:

$$Ar = \frac{Dx}{Dy} \qquad \text{Eq. (2-34)}$$

where *Dx* is the dimension of the fault in the *X* direction and *Dy* is the dimension of the fault in the *Y* direction. It must be recalled that, when the treat as fault behavior option is selected, the area source must have exactly four vertexes that form a rectangle that lies in a single plane. By definition, the *X* direction is the one that joins vertexes 1 and 2 of the area source, while the *Y* direction is the one that joins vertexes 2 and 3 as shown in Figure 2-5.

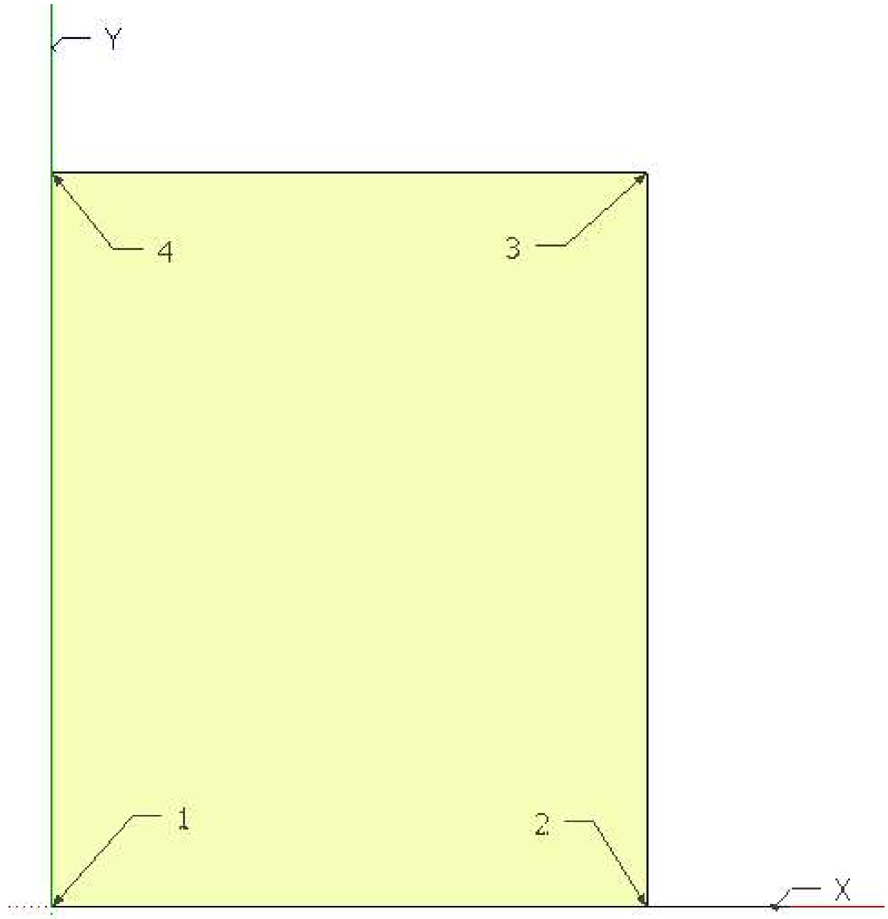

Figure 2-5 Definition of an area source with the treat as fault behavior option

Elliptical ruptures are constructed with the aspect ratio indicated by the user until they do not fit in the rectangular area of the source with that aspect ratio to accommodate the largest possible rupture area. When this situation is reached, R-CRISIS has a smooth transition between the aspect ratio given by the user and the rectangular area source aspect ratio (i.e. width/length). In other words, for small magnitudes, rectangular ruptures start having the aspect ratio indicated by the user, but the aspect ratio might change as magnitude increases, approaching smoothly the rectangular area aspect ratio width/length. Note that this issue slightly can affect the estimation of R_{RUP} and R_{JB} distances for relatively large earthquakes.

Note: An area source with treat as fault behavior is equivalent to a source modelled as a rectangular fault.

Breaks always

When this behavior option is selected, at the source, regardless of the magnitude, the area will break completely for each earthquake. This option is normally used for earthquakes which, by hypothesis, will completely fill up the rupture area, regardless their magnitudes. In view of this, there is only one hypocenter associated to the area. This hypocenter is the point within the source closest to the computation site. Again, this is only relevant when R_{RUP} or R_{JB} are being used as distance measures.

Note: in this case, the values of *K1* and *K2* coefficients provided to R-CRISIS become irrelevant.

Leaky and strict boundaries

As mentioned before, depending on the selection of the behavior for the seismic sources, it is possible to allow the ruptures to extend beyond its boundaries or be always within the plane. The first case is known as leaky boundary and epicenters can occur at the edges of the sources as shown in Figure 2-6. In this case, L corresponds to *Dx* whereas W corresponds to *Dy*.

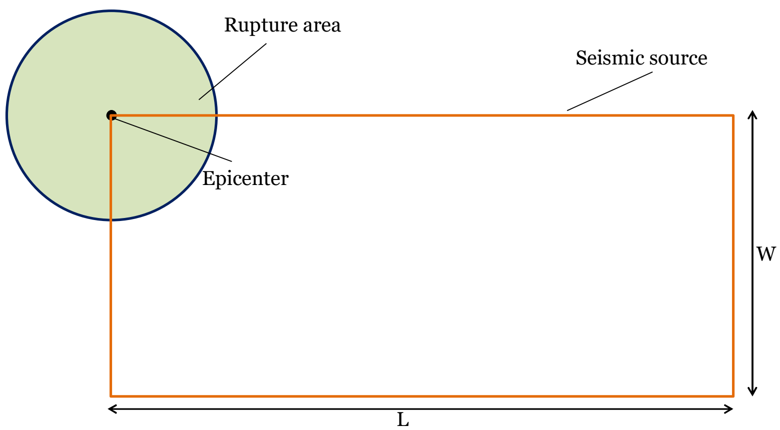

Figure 2-6 Schematic representation or the leaky boundary behavior

In the second case, known as strict boundary, the geometry of the rupture is not allowed to extend beyond the geometric limits of the source and then, depending on the size of the rupture, the location of the epicenter is adjusted so that the totality of the rupture can be accommodated within the plane as shown in Figure 2-7. In this case, L corresponds to *Dx* whereas W corresponds to *Dy*.

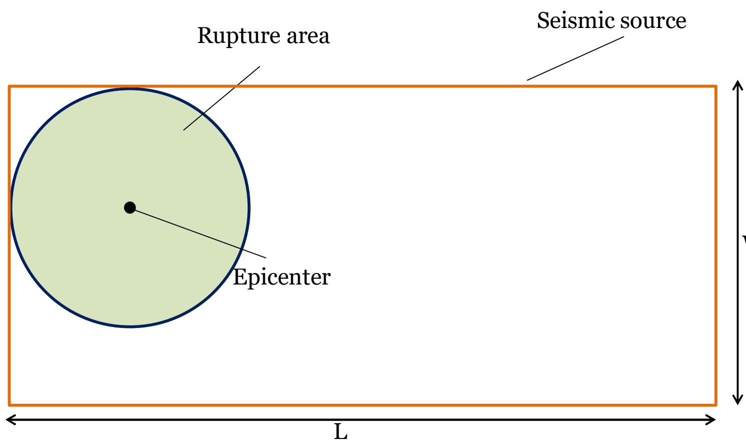

Figure 2-7 Schematic representation or the strict boundary behavior

2.2.2 Area plane sources

This geometry model considers the active source in the same way as an area source, explained before, with the differences that for this case the rupture planes can have an orientation defined by the user. They are different from the common area sources because in said geometry model the ruptures are planes formed by the area itself, whereas in this geometry model, the rupture planes have a constant orientation provided to R-CRISIS by the user. The geometry of the source (plane coordinates and depth) is defined in the same way as in the area case.

Orientation of the rupture plane

The orientation of the rupture planes of the area plane sources are assigned by means of the values provided to R-CRISIS by the user for the strike (in degrees) and the dip (in degrees). Figure 2-8 shows three examples with the same strike and different dip values, as understood by R-CRISIS (values in parenthesis indicate the normal vectors associated to the different orientations).

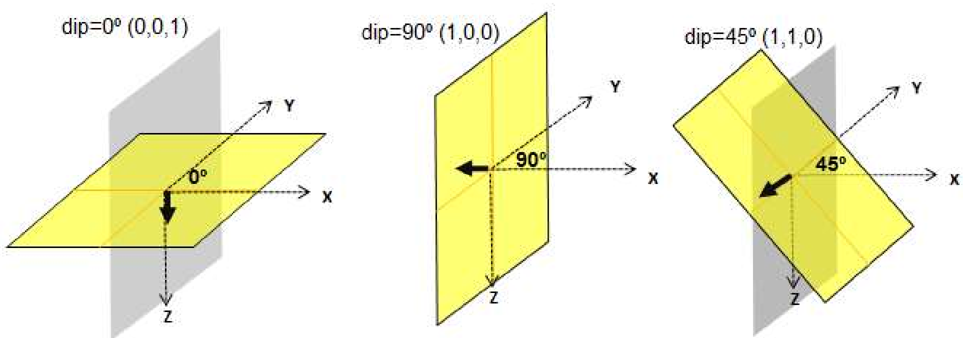

Figure 2-8 Example of dip values to orientate the rupture planes in R-CRISIS

Size of the rupture

A magnitude-dependent size of the rupture plane can be assigned using parameters *K1* and *K2*. This choice is, again, relevant only in the cases in which R_{RUP} or R_{JB} are used as distance measures. The way in which R-CRISIS recognizes those values associated to the size of the rupture is the same as explained for the case of the area sources. Figure 2-9 shows schematically how, at one sub-source, rupture areas associated to different *M* values are considered when this geometry model is used. The grey plane corresponds to the area source whereas the yellow plane corresponds to the orientation of the rupture provided by the user by means of the unit vector.

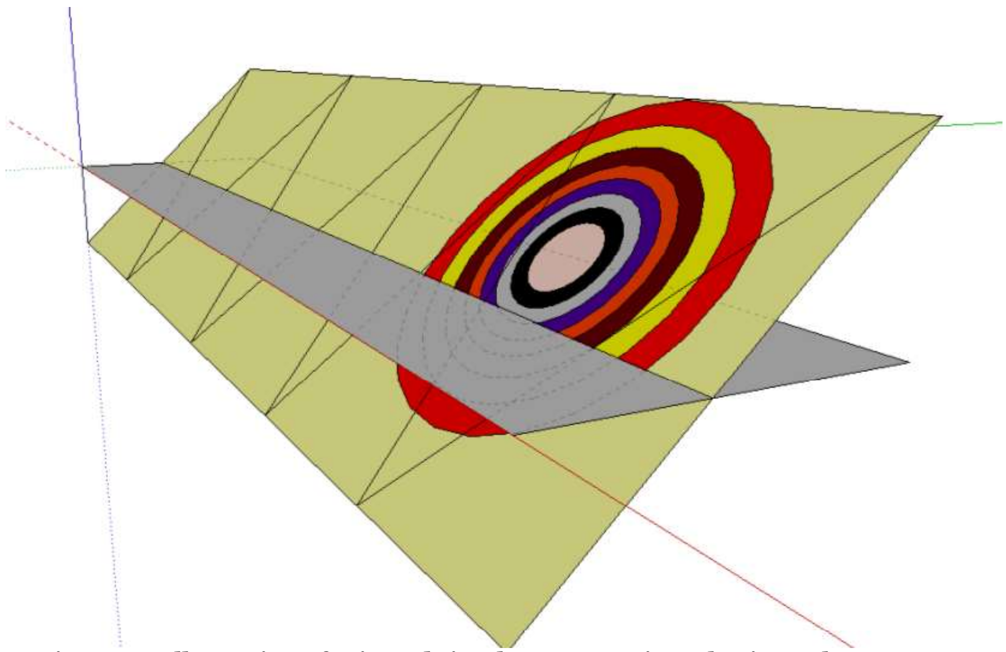

Figure 2-9 Illustration of oriented circular ruptures in an horizontal area source

Aspect ratio

The same approach as in the case of area sources is followed. *Dx* is understood by R-CRISIS in the direction of the strike whereas *Dy* in the direction of the dip.

2.2.3 Volume sources

In R-CRISIS the seismic sources can be treated as volumes by first defining the geometry of an area source and then setting the thickness of the volume and the number of slices in which the seismicity is to be distributed. This means that the volume source is modelled by *N* area sources (slices), all with the same coordinates but located at different depths as shown in Figure 2-10. The yellow polygon represents the original area source and the grey polygons represent the additional slices that comprise the volume area. In this case, the seismicity is evenly divided among the *N* slices (4 in the case of Figure 2-10). The option is intended to simulate an even distribution of seismicity with depth.

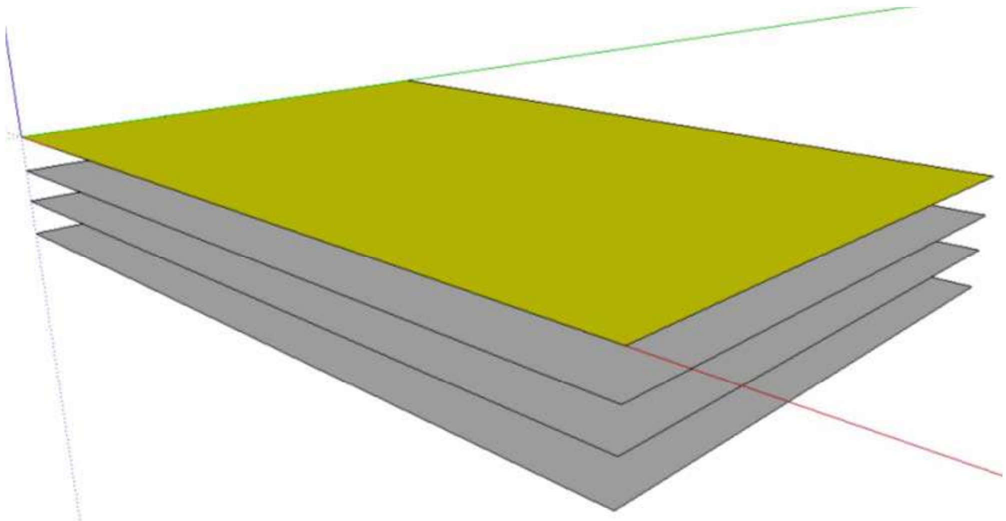

Figure 2-10 Volume sources in R-CRISIS

The seismicity models to be used when this geometric representation is chosen are the modified G-R, the characteristic earthquake, the generalized Poisson and the generalized non-Poisson. In all cases, the seismicity rates (λ) are uniformly distributed into the N slices. That is, each slice has a seismicity rate equal to λ/N but located at a different depth.

Note: if N=1, the source will be considered by R-CRISIS as an area source.

2.2.4 Line sources

This geometry model allows defining the active source as a fault (line) source. Line sources are, in general, polylines defined by the 3D coordinates of their vertexes. Figure 2-11 shows a fault source of 4 vertexes, located in the XZ plane with varying depth.

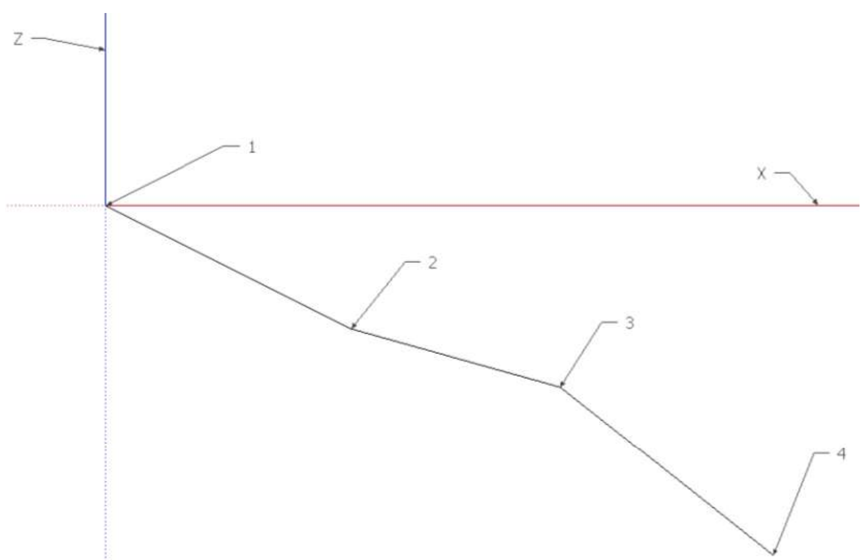

Figure 2-11 Example of a fault area with varying depth and 4 vertexes

Note: the "break always" behavior option for line sources works exactly in the same way as in the case of area sources.

Relation between magnitude and rupture length

For line sources, R-CRISIS relates the rupture length, *L*, to the magnitude *M*, for surface rupture length (SLR) and subsurface rupture length (SSLR) by means of:

$$L = K_3 \cdot e^{K_4 M} \qquad \text{Eq. (2-35)}$$

where *L* is in km and K_3 and K_4 are coefficients that relate the magnitude with the length of the rupture. For instance, the regression form proposed by Wells and Coppersmith (1994) has the following form:

$$\log L = a + bM \qquad \text{Eq. (2-36)}$$

Equation 2-36 can be rewritten as:

$$L = 10^a \cdot 10^{bM} \qquad \text{Eq. (2-37)}$$

As in the case of the area sources, equations 2-35 and 2-37 have a similar structure that allows the following equivalences:

$$K_3 = 10^a \qquad \text{Eq. (2-38)}$$

$$e^{K_4} = 10^b \qquad \text{Eq. (2-39)}$$

Tables 2-9 to 2-11 show the regression coefficients, the R-CRISIS coefficients and the equivalences for the Wells and Coppersmith (1994) model.

Table 2-9 Wells and Coppersmith (1994) SRL and SSRL rupture length regression coefficients

Model	*a*	*b*
Strike-slip (SLR)	-3.55	0.74
Reverse (SLR)	-2.86	0.63
Normal (SLR)	-2.01	0.50
All (SLR)	-3.22	0.69
Strike-slip (SSLR)	-2.57	0.62
Reverse (SSLR)	-2.42	0.58
Normal (SSLR)	-1.88	0.50
All (SSLR)	-2.44	0.59

Table 2-10 R-CRISIS SRL and SSRL rupture length coefficients for the Wells and Coppersmith (1994) model

Model	K_3	K_4
Surface Rupture Length (SLR) - Strike-slip	0.00028	1.70391
Surface Rupture Length (SLR) - Reverse	0.00138	1.45063
Surface Rupture Length (SLR) - Normal	0.00977	1.15129
Surface Rupture Length (SLR) - All	0.00060	1.58878
Subsurface Rupture Length (SSLR) - Strike-slip	0.00269	1.42760
Subsurface Rupture Length (SSLR) - Reverse	0.00380	1.33550
Subsurface Rupture Length (SSLR) - Normal	0.01318	1.15129

Table 2-11 Equivalences between Wells and Coppersmith (1994) and R-CRISIS coefficients for SLR and SSLR

Model	Eq 2-38		Eq 2-39	
	10^a	K_3	10^b	e^{K_4}
Strike-slip (SLR)	2.82E-04	2.80E-04	5.495	5.495
Reverse (SLR)	1.38E-03	1.38E-03	4.266	4.266
Normal (SLR)	9.77E-03	9.77E-03	3.162	3.162
All (SLR)	6.03E-04	6.00E-04	4.898	4.898
Strike-slip (SSLR)	2.69E-03	2.69E-03	4.169	4.169
Reverse (SSLR)	3.80E-03	3.80E-03	3.802	3.802
Normal (SSLR)	1.32E-02	1.32E-02	3.162	3.162
All (SSLR)	3.63E-03	3.63E-03	3.890	3.890

In the case of line sources, R-CRISIS assumes that the earthquakes occur along a line defined by the source geometry, and that the rupture length will be centered at the hypocenter as shown in Figure 2-12.

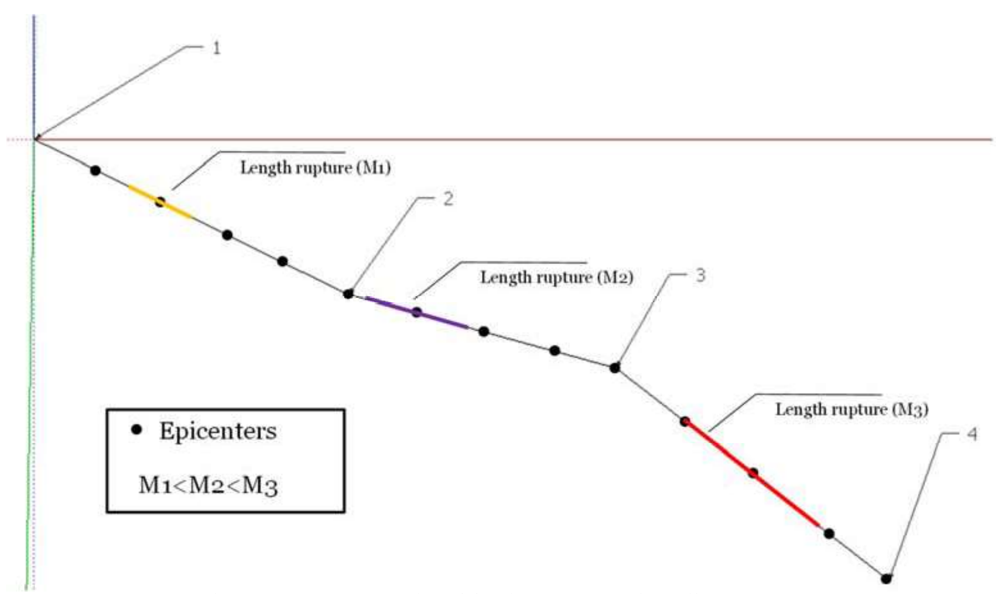

Figure 2-12 Example of fault ruptures in a line source

2.2.5 Point sources

This option defines the active source as a collection of point sources, in which each vertex is assumed to be in R-CRISIS an individual point source. Each point is a potential earthquake hypocenter and is defined by means of the following parameters:

1. Longitude, latitude and depth (in km) of the point.
2. A unit vector normal to the rupture plane associated to each point source. This unit vector is relevant only when the GMPE associated to this source uses distance measures for which the rupture areas are relevant (i.e. R_{RUP} or R_{JB}).

Since point sources are generally used to geometrically describe potentially thousands of focal locations, information about this type of source is provided by the user to R-CRISIS by means of an ASCII file with extension *.ssg, with the structure shown in Table 2-12.

Table 2-12 Point geometry file structure

Point geometry file		
Description	**Variable**	**Type**
ID Header	Header	String
Number of point sources	TotSrc	Integer
Geometry record for source 1	Geom(1)	Geometry record
Geometry record for source 2	Geom(2)	Geometry record
....	...	...
Geometry record for source TotSrc	Geom(TotSrc)	Geometry record

Table 2-13 on the other hand describes the structure of a geometry record.

Table 2-13 Geometry record file structure

Geometry record		
Description	**Variable**	**Type**
Hypocentral location	h.X	in degrees
	h.Y	in degrees
	h.Z	in km (positive)
Unit vector describing the orientation of the fault plane	e1.x	These three values describe a unit vector normal to the fault plane. X is longitude, Y is latitude and Z is depth
	e1.y	
	e1.z	

Finally, Table 2-14 shows an example of a point-source geometry file, where *N* point sources are geometrically described:

Table 2-14 Point-source geometry file example

Line in file	Comment
Header	Header line for identification purposes
N	Number of points described
Long1 Lat1 Dep1 0 0 0	Each line provides the longitude, latitude and depth for the N point sources. In tis case the coordinates of the unit vector normal to the fault plane is 0,0,0 which means that they are unknown or irrelevant. Those are relevant for instance if an attenuation model based on focal distance is to be used. If the unit vector normal to the fault plane is described with (0,0,0) a horizontal plane will be default
Long2 Lat2 Dep2 0 0 0	
....	
....	
....	
....	
....	
....	
LongN LatN DepN 0 0 0	

As explained in the case of area sources, the relation between the magnitude and the rupture area size depends on M and the $K1$ and $K2$ parameters and for this geometry model is treated in the same way than for the area sources in R-CRISIS.

One special case of point sources corresponds to the use of a stochastic event catalogue (SEC) that is to be arranged in *.csv format with the following fields:

- ID (string value)
- Rupture area (in km^2)
- Annual probability
- Magnitude
- Strike
- Dip
- Rake
- Longitude
- Latitude
- Depth
- Aspect ratio

The strike angle is measured in the same way as the azimuth; the dip angle is measured in clockwise order with reference to the strike angle. The dip angle is always ≤ than 90º (if a higher angle is required, it needs to be modified by 180º). The length of the rupture, L, is measured in the strike direction whereas its width, W, is measured in the dip (down-dip) direction. Aspect ratio is therefore, equal to L/W.

Note: each SEC is treated in R-CRISIS as a source so the same GMPE will be used for all events included in it.

2.2.6 Gridded sources

This option defines the active source as a collection of point sources located at the nodes of a rectangular grid that is parallel to the surface of the Earth (i.e. a grid in which all the nodes have the same depth). Each one of the nodes is considered in R-CRISIS as a potential hypocenter. The nodes of the grid are the only hypocenters that R-CRISIS will consider in the calculations as point sources. If the grid is not sufficiently dense, the modelled sources may be too far apart and may not suitable for performing a good PSHA.

The grid is defined by the parameters shown in Table 2-15 which construct it in the way the grid shown in Figure 2-13.

Table 2-15 Required parameters for the definition of a grid source

Description	Longitude	Latitude
Origin (Usually the SW corner)	Xmin	Ymin
End (Usually the NE corner)	Xmax	Ymax
Number of lines in each orthogonal direction	*N*	*M*

After this, the total number of nodes in the grid is equal to $N*M$.

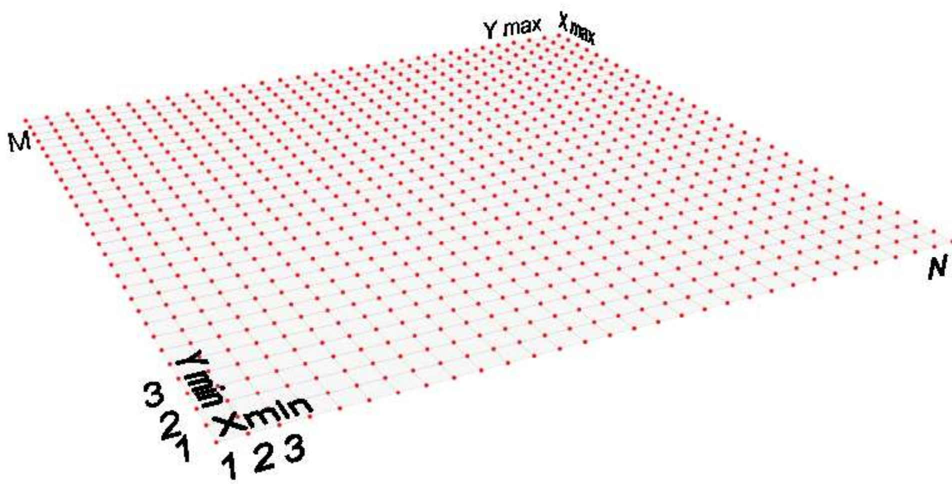

Figure 2-13 Basic grid parameters

The seismicity model that can be used together with this geometry model is the modified G-R where it is considered that M_0 is constant across the seismic province but λ_0, β and M_U can have geographical variations defined by means of separate grids, one for each of these parameters. The values of those parameters are provided to R-CRISIS through 3 different files with *.grd format (Surfer 6 ASCII or binary). Figure 2-14 shows a schematic representation of the structure of this model. Those denoted as L0.grd, EB.grd and MU.grd correspond to the λ_0, β and M_U grids.

Note: the uniform depth of the seismicity grid is provided to R-CRISIS in kilometers.

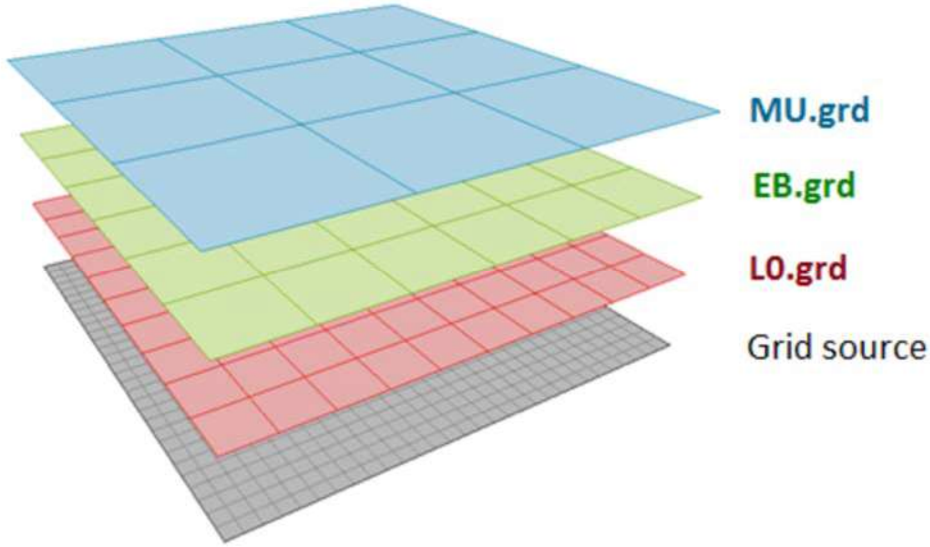

Figure 2-14 Seismicity parameters structure for the gridded geometric model

Note: the limits (Xmin, Xmax, Ymin and Ymax) of each seismicity parameters' grid must coincide with the ones of the source geometry grid but the number of rows and columns in them can be equal or smaller than those of the seismicity grid. Even more, the number of rows and columns may be different for the three seismicity parameters.

The relation between the magnitude and the rupture area size again depends on *M* and the *K1* and *K2* parameters. For the gridded geometry model, those are treated in the same way than for the area sources in R-CRISIS.

Delimitation polygon (optional)

The grid can be delimited by a polygon or group of polygons provided in Shapefile *.shp format as schematically shown in Figure 2-15. Only the grid nodes that lie within at least one of the polygons will be considered active point sources.

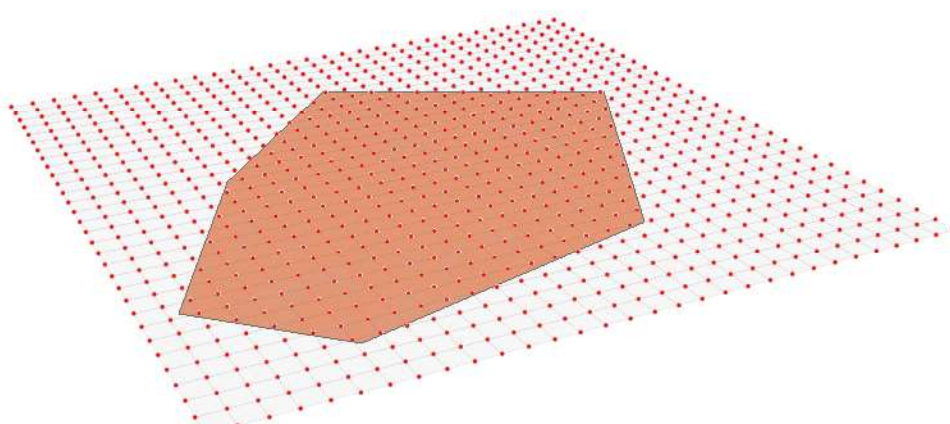

Figure 2-15 Schematic representation of a delimitation polygon

Orientation of rupture plane (optional)

The orientation of the rupture planes can be provided for the grid sources to R-CRISIS by defining normal vectors as schematically shown in Figure 2-16. For this geometry model, these vectors are provided to R-CRISIS by means of three grids that contain the X, Y and Z values, respectively, of the unit vectors that define the plane orientations. These files must be in *.grd format (either Surfer 6 ASCII or Surfer 6 Binary formats) and have the same resolution for the X, Y and Z values than the gridded seismic source.

The names of these files are fixed and are as follows:

- NormalVector_X.grd
- NormalVector_Y.grd
- NormalVector_Z.grd

The path of the folder containing these files must be provided to R-CRISIS. If normal vector grids are not provided to R-CRISIS, horizontal rupture planes (dip=0°) are assumed. Normal vector grids must have the same origin, end and spacing than the main source grid:

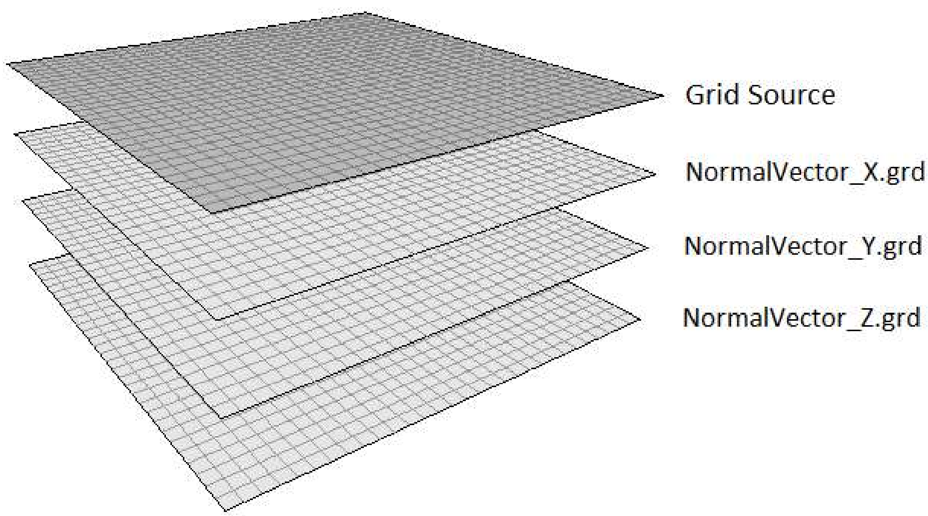

Figure 2-16 Structure of input data to define the orientation of ruptures in the gridded model

The inclusion of normal vector grids is relevant only in the cases in which R_{RUP} or R_{JB} are used as distance measures in the attenuation relations and also in those cases where rupture areas are different from 0 (i.e. parameters *K1* and *K2*>0).

2.2.7 Rectangular faults

This geometry defines a rectangle in which hypocenters can take place, without allowing rupture areas to be partially out of the rectangle (*strict boundary*). This rectangle is a common model for an earthquake fault and it is defined by the following parameters:

Upper lip or fault trace

This line, defined by at least two points, describes the projection of trace of the fault on the Earth's surface. The distance between the two points that form the strike line is the length of the fault, and the angle they form defines its strike. Both points of the strike line must have the same depth, which marks the beginning of the seismogenic zone as shown in Figure 2-17.

Width

This parameter defines the dimension of the fault in the direction perpendicular to the strike line, as shown in Figure 2-17.

Dip

This value defines the dip angle (in degrees) of the fault. This angle must be between 0° (horizontal fault) and 90° (a vertical fault as in Figure 2-17). Negative dip values are not accepted by R-CRISIS and therefore, if required, the strike must be modified by 180°.

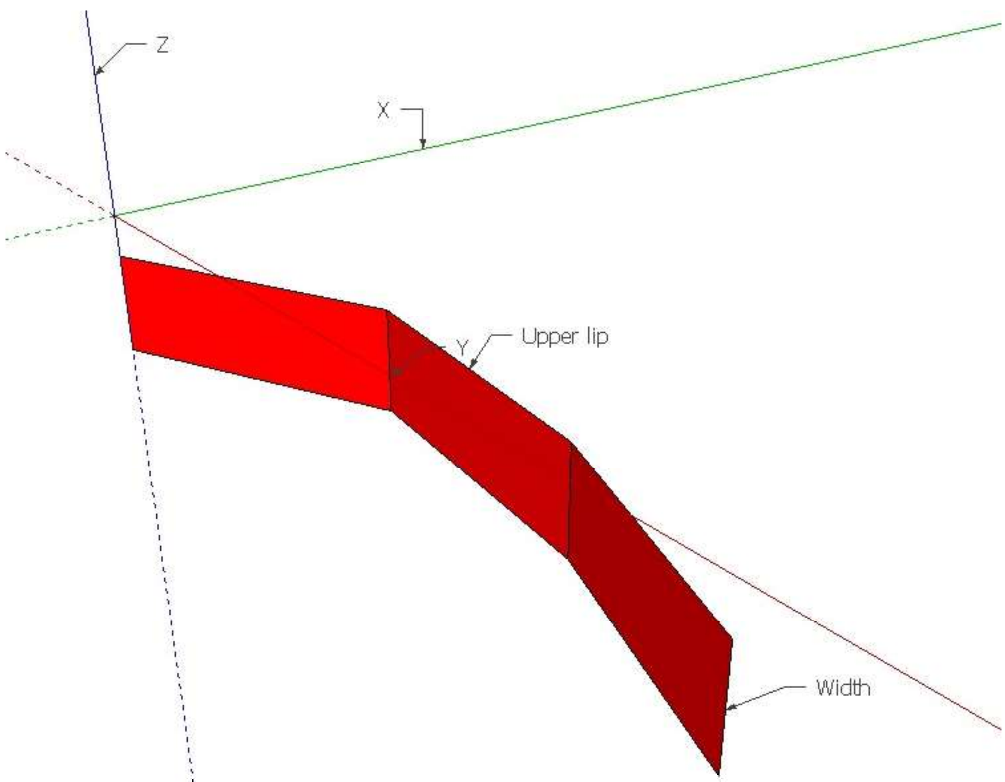

Figure 2-17 Example of a rectangular fault with dip equal to 90°

Note: *K1* and *K2* parameters as well as the fault aspect ratio are defined in the same way as in the case of area sources described before.

Stirling fault

There are several possibilities to resolve the geometry of the lower lip of a bending fault. If this option is selected, then the fault will be considered an Stirling fault, in which the upper lip and the average dip are used to create a corrugated surface by translating the upper lip down dip, perpendicular to the average fault strike. If this option is not selected, then the fault is treated as a Frankel fault, where the dip direction of each rectangle is perpendicular to the strike of its local segment. For relatively smooth bending, there is little difference between both types of fault.

2.2.8 Slab geometries

This geometry model can be used to represent in-slab sources where, instead of using the area geometry model and assuming that the ruptures are points occurring within the plane defined by the user, using the geometry of the top end of the slab a set of rectangular faults are generated and ruptures therefore occur on them.

This geometry generates a seismogenetic source from a polygon that needs to have the nodes defined in the way shown in grey in Figure 2-18. Segment 1-2 corresponds to the upper lip of the slab whereas segment 3-4 corresponds to its lower lip. The depth (in km) of nodes 1 and 2 needs to be equal and the same condition holds for the depth of nodes 3 and 4. With these input data, a set of rectangular faults (blue) is generated, as shown in Figure 2-18 after defining 3 slices (rectangular faults).

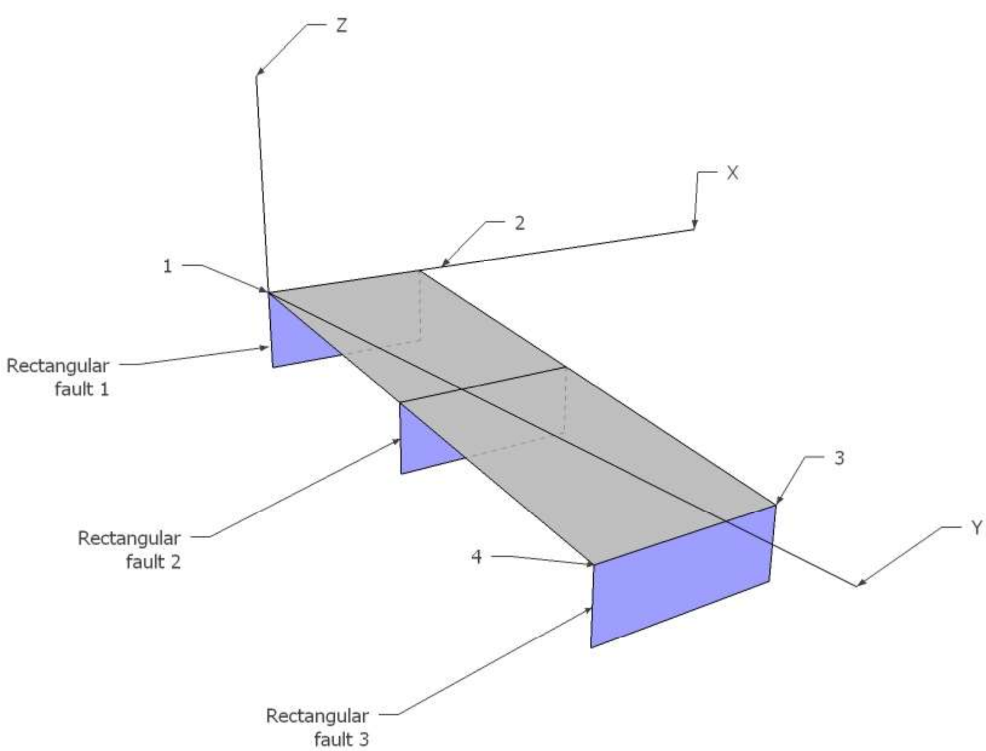

Figure 2-18 Illustration of slab geometry model in R-CRISIS

Additionally, the following parameters need to be defined by the user:

Dip

This line, defined by two points, describes the projection of trace of the fault on the Earth's surface. The distance between the two points that form the strike line is the length of the fault, and the angle they form defines its strike. The same dip applies to all rectangular faults in which the slab is divided.

Width

This parameter defines the dimension of the fault in the direction perpendicular to the strike line. The same width applies to all rectangular faults in which the slab is divided.

Note: If the dip is set to 90°, the width would correspond then to the thickness of the slab.

Rectangular ruptures

This parameter indicates if the ruptures will be considered as rectangular (true) or elliptical (false). The same choice applies to all rectangular faults in which the slab is divided.

Note: *K1* and *K2* parameters as well as the fault aspect ratio are defined in the same way as in the case of area sources described before.

2.2.9 Ruptures

In R-CRISIS it is also possible to describe the occurrence of future earthquakes by means of ruptures for which several characteristics, as explained herein, are defined. This is an approach that can be also used for validation purposes if only a historical catalogue is used.

Each rupture needs to have assigned information about the following parameters:

- Date (DD/MM/YY)
- Area (Km^2)
- Annual occurrence probability
- Magnitude
- Strike
- Dip
- Rake
- Longitude (Decimal degrees)
- Latitude (Decimal degrees)
- Depth (Km)
- Aspect ratio

The information for each set of ruptures needs to be provided in terms of a *.csv file. Each *.csv file is considered by R-CRISIS as a seismic source for which a GMPE needs to be assigned.

2.3 Measuring distances in R-CRISIS

Distances in R-CRISIS are estimated using the coordinate system known as World Geodetic System 84 (WGS84) that allows locating any site within the Globe by means of three values. To facilitate the use of R-CRISIS in different locations, this coordinates system has been selected since it is the only one that is used and valid at global level. The geometry of the sources as well as the location of the computation sites are provided to R-CRISIS using decimal degrees and those distances are converted to kilometers using by assuming that the Earth is a sphere with radius equal to 6366.707 km. This distance corresponds to the average value of the major and semi-minor axis of the WGS84 datum (Department of Defense, 1997).

In R-CRISIS, there are four ways of measuring site-to-source distances:

1. Focal distance (R_F)
2. Epicentral distance (R_{EPI})
3. Joyner and Boore distance (closest distance to the projection of the fault plane on the Earth's surface; R_{JB})
4. Closest distance to rupture area (R_{RUP})

Figure 2-19 illustrates the differences between the measure distances recognized by R-CRISIS considering that H corresponds to the focal depth.

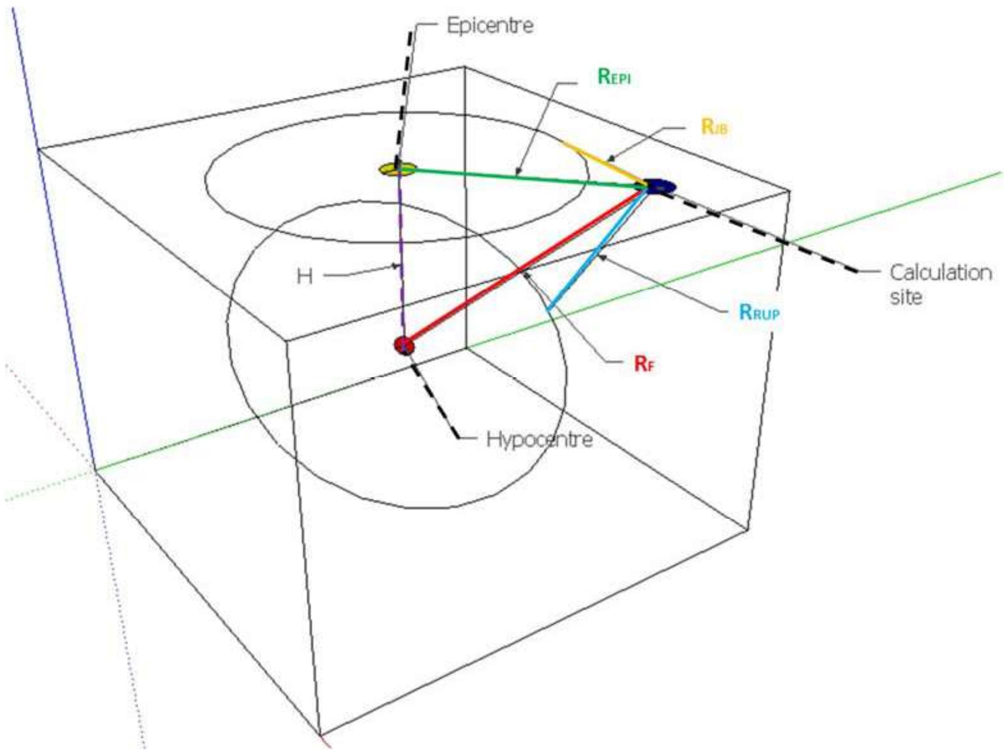

Figure 2-19 Distance measures implemented in R-CRISIS

Computation of R_F and R_{EPI} deserves no further comments but, computation of R_{RUP} and R_{JB}, however, requires the specification of a rupture area (or length). In R-CRISIS, as explained before, the area is assumed to be circular, with radius *r*, which depends on magnitude *M* together with the *K1* and *K2* parameters. The circular rupture is contained in the plane defined by the triangle resulting from source subdivision (see Section 2.6.1), whose centroid is assumed to be the hypocentral location (see Figure 2-4).

Note: if the site is within the projection of the fault in the Earth's surface, R_{JB}=0 and R_{RUP}=H.

The user must indicate R-CRISIS what type of distance is to be used within the PSHA, which in most cases depends on the characteristics of the GMPE being used. For elliptical and rectangular ruptures, R_{RUP} and R_{JB} are computed in an exact and rigorous manner within the distances of interest between the rupture and each calculation site. When the ratio between the rupture radius and R_F or R_{JB} is smaller than 0.025, R-CRISIS performs the following approximation: R_{RUP}=R_F or R_{JB}=R_{EPI}. This approximation has little, if any implications, in the final results, even for large magnitudes.

2.4 Strong ground motion attenuation models

In general, ground motion prediction equations (GMPE), also referred to as attenuation relations, establish probabilistic relations between earthquake characteristics, intensities and distances at the computation sites. These relations are probabilistic since, for given earthquake characteristics, the intensities are regarded as random variables whose probability distribution is completely fixed by the GMPE. In most of the cases this means that at least the first two probability moments (e.g. the median and the standard deviation of the natural logarithm in the lognormal case) of the probability distribution must be defined for the GMPE. R-CRISIS recognizes three different "families" of GMPE (i.e. the way in which those are included in the seismic hazard analysis project):

1. GMPE tables: In these tables, relations between earthquake characteristics and intensities at a site are given in terms of the following parameters: magnitude, structural period, source to site distance and depth. For the first probability moment (usually the median of a lognormal distribution), the attenuation relations are matrices in which the rows account for the magnitudes and the columns account for the distances. Note that when using attenuation tables, the relations between magnitude, distance and intensity do not need to be of parametric nature, since the intensity medians are given, point by point, for the different magnitude-distance combinations.
2. Built-in GMPE: These are popular models, published in the literature and developed by well-known authors, in which magnitudes, distances and intensities are probabilistically related by, usually, a set of formulas or parametric equations. There is a set of built-in models ready to use in R-CRISIS and there is also the possibility of adding new models. See Table 2-17 for the list of built-in GMPM available in R-CRISIS.
3. Generalized models: Generalized attenuation models are non-parametric probabilistic descriptions of the ground motions produced by an earthquake. In the framework of R-CRISIS, a generalized attenuation model is a collection of probabilistic footprints, one for each of the events considered in the analysis. Each footprint provides, in probabilistic terms, the geographical distribution of the intensities produced by this specific event.
4. Hybrid models: Hybrid models, sometimes known also as "composite" models, are lineal combinations of other types of GMPE, either user given or built-in. Sometimes, and for some applications, they can be used to replace, to some extent, logic trees.

A detailed description of each of these families is presented next.

2.4.1 GMPE tables

These tables provide R-CRISIS the probabilistic relations between magnitude, source-site distance and intensities. Each attenuation table must be saved in a different file and must have the structure explained next.

Attenuation table header

All the lines of this portion of the file are optional. The user, however, must be aware of the default values that are used for the parameters that are described herein. The header can contain up to 4 lines that provide different characteristics of the attenuation table and lines can be given in any order. Field names (including capital letters) are fixed. Table 2-16 describes the four possible header fields recognized by R-CRISIS.

Table 2-16 Description of the header fields accepted by R-CRISIS for attenuation tables

Field name	Field value	Comments	Default value
Description	A string providing a brief description of the attenuation table (e.g. author, date of publication, suitable tectonic environment, etc.)	This information is for displaying on the "Attenuation data" screen	"Not available"
Units	A string providing the units for whith the model was developed for	The original units are displayed for information purposes and will guide the user to define if a units coefficient is needed	"Not available"
Distribution	An integer number indicating the probability distribution assigned to the residuals of the attenuation model	Supported values are: Normal = 1, Lognormal = 2, Beta = 3, Gamma = 4	2 (Lognormal)
Dimension	A string value providing the physical dimension of the intensities described in the attenuation table	See Table 2.16	"Acceleration"

Parameters defining the magnitude limits (1 line)

The values defining the magnitude limits are provided in one line and denoted as: *MINF, MSUP, NMAG* as described in Table 2-17.

Table 2-17 Description of magnitude range and number in attenuation tables

Variable	Description
MINF	Lower limit of magnitude given in the table
MSUP	Upper limit of magnitude given in the table
NMAG	Number of magnitudes for which intensity is given

R-CRISIS assumes than intensities are given for magnitudes *M(K)*, where

$$M(K) = MINF + (K-1) * DMAG \quad \text{Eq. (2-40)}$$

and,

$$DMAG = \frac{(MSUP - MINF)}{(NMAG - 1)} \quad \text{Eq. (2-41)}$$

Parameters defining the distance limits and type (1 line)

The values defining the distance limits (and type) are provided in one line and denoted as: *RINF, RSUP, NRAD, TYPE* and described in Table 2-18.

Table 2-18 Description of distance range, number and type in attenuation tables

Variable	Description
RINF	Lower limit of distance given in the table
RSUP	Upper limit of distance given in the table
NRAD	Number of distances for which intensity is given
TYPE	An integer indicating the type of distance used by the attenuation table

R-CRISIS assumes that intensities are given for distances *R(K)*, where

$$\mathrm{Log}(R(K)) = \mathrm{Log}(RINF) + (K-1) * DLRAD \quad \text{Eq. (2-42)}$$

and

$$DLRAD = \frac{(\mathrm{Log}(RSUP) - \mathrm{Log}(RINF))}{(NRAD-1)} \quad \text{Eq. (2-43)}$$

which means that distances are logarithmically spaced.

The *TYPE* field can have any of the values shown in Table 2-19[8], depending on the type of distance for which the GMPE has been developed.

Table 2-19 Codes for types of distances in attenuation tables

Value	Type of distance
1 (or blank)	Focal (R_F)
2	Epicentral (R_{EPI})
3	Joyner and Boore (R_{JB})
4	Closest to rupture area (R_{RUP})

[8] Colors indicate the distance type in Figure 2-15

Parameters defining the spectral ordinate, standard deviation, hazard intensity and depth coefficient

Once the magnitude and distance ranges and limits have been defined in each attenuation table, the following values are required for each spectral ordinate in the same line. For notation purposes, the main data of these lines (one for each spectral ordinate) are referred to as: *T(J), SLA(J,0), AMAX(J), COEFH* which complete description is provided in Table 2-20.

Table 2-20 Description of attenuation table data

Variable	Description
T(J)	Structural period of the jth spectral ordinate. It is used only for identification purposes and to plot the uniform hazard spectra, so in the cases in which structural period has no meaning, it can be a sequential numbe
SLA(J,0)	Standard deviation of the natural logarithm of the jth measure of intensity. A value of *SLA(J,0)* ≤ 0 implies that the user will provide standard deviations that vary with magnitude. In this case, the corresponding σ values (one for each or the *NMAG* magntudes) has to be given after the table of *SA()* values
AMAX(J)	See Section 2.4.2 for the definition of this value
COEFH	Depth coefficient (see below)

Some recent GMPE include a coefficient to make the intensity explicitly dependent on the focal depth. This information can be provided by the user to R-CRISIS by means of the *COEFH* coefficient, so that:

$$MED(A \mid M, R) = Sa(M, R) \cdot \exp(COEFH * H) \quad \text{Eq. (2-44)}$$

where *MED*(*A*|*M,R*) is the (depth-dependent) median value of intensity for given values of magnitude *M* and distance *R* and *Sa*(*M,R*) corresponds to the median intensity given in the attenuation table for the same values of magnitude and distance, and *H* is focal depth.

Matrix of median intensities, associated to a magnitude (row) and a distance (column)

For each spectral ordinate the attenuation table includes a matrix that contains the median intensities associated to the magnitudes (rows) and to the distances (columns). For notation purposes those are referred to as: *Sa(1,1,1), Sa(1,1,2),…,Sa(J,K,L),….,Sa(NT,NMAG,NRAD)* where *Sa(J,K,M)* corresponds to the median value of the intensity, for the J^{th} spectral ordinate, the K^{th} magnitude and the L^{th} distance.

Only if *SLA(J)* <=0:

SLA(J,1)
SLA(J,2)
…
SLA(J,NMAG)

Note: the attenuation tables to be used in R-CRISIS are to be saved in ASCII format and with *.atn extension.

Example of a *.atn file

Table 2-21 shows an example of an attenuation table that includes *NT*=2 periods (or intensity measures). Values shown in black are those to be included in the table whereas those shown in red provide only a description of the meaning of the values used in this example.

Table 2-21 Example of a *.atn file (user defined attenuation table)

#	: Description	Example of attenuation table (CRISIS2015 manual)						
#	: Units	gal						
#	: Distribution	2						
#	: Dimension	Spectral acceleration						
4.5	8.5	5		5 magnitudes between 4.5 and 8.5				
5.0	500.0	10	1	10 distances between 5 and 500 km (log-spaced); focal distance				
0.0	0.7	0.0	0.0	Period 0; σ=0.7 , Amax=0 (no truncation), CoefH=0				
119.3	97.5	70.5	45.3	14.7	7.6	3.4	1.2	0.3
202.5	165.0	120.1	76.9	24.3	12.6	5.8	1.8	0.5
344.0	251.2	201.5	130.6	43.5	22.3	9.8	3.0	0.8
584.1	477.4	354.3	221.8	72.5	36.4	16.5	5.6	1.3
992.0	811.2	585.6	376.7	122.5	60.1	27.5	9.6	2.4
0.5	-1.0	0.0	0.0035	Period 0.5; σ variable with M, Amax=0 (no truncation), CoefH=0.0				
239.4	217.6	190.6	165.4	134.8	127.7	123.5	121.3	120.4
322.6	285.1	240.2	197.0	144.4	132.7	125.9	121.9	120.6
464.1	371.3	321.6	250.7	163.6	142.4	129.9	123.1	120.9
704.2	597.5	474.4	341.9	192.6	156.5	136.6	125.7	121.4
1112.1	931.3	705.7	496.8	242.6	180.2	147.6	129.7	122.5
0.83				5 values of magnitude-dependent σ (one for each magnitude)				
0.78								
0.62								
0.63								
0.51								

Physical dimensions of the hazard intensities

To have stricter checks of the compatibility among different GMPE when performing logic-tree computations (see Section 2.12), each GMPM must be assigned a physical dimension of the measures of hazard intensity that the model is describing. The physical dimension of most GMPE is spectral acceleration (because they are usually constructed for PGA and the response spectral ordinates at selected fundamental periods), but other physical dimensions are also accepted and can be used. R-CRISIS accepts the physical dimensions shown in Table 2-22, which correspond to classes defined for this purpose.

Table 2-22 Physical dimensions accepted by R-CRISIS

Physical dimension	Assembly name
Acceleration	Crisis2008.NewAttenuation.dll
Velocity	Crisis2008.NewAttenuation.dll
Displacement	Crisis2008.NewAttenuation.dll
MMI	Crisis2008.NewAttenuation.dll
MCSI	Crisis2008.NewAttenuation.dll
DuctilityDemand	ExtraDimensions.dll
ISDrift	ExtraDimensions.dll

Although only these physical dimensions are recognized by R-CRISIS, it is relatively simple to construct additional classes associated to other intensity measures. To do so, the constructed class must implement the methods shown in Table 2-23.

Table 2-23 Implemented methods for physical dimensions in R-CRISIS

Method	Purpose
Public ReadOnly Property distancePow() As Integer	Returns an integer indicating the distance power of this dimension
Public ReadOnly Property forcePow() As Integer	Returns an integer indicating the force power of this dimension
Public ReadOnly Property timePow() As Integer	Returns an integer indicating the time power of this dimension
Public ReadOnly Property chargePow() As Integer	Returns an integer indicating the charge power of this dimension
Public MustOverride ReadOnly Property name() As String	Provides a number specific to the class
Public Overrides Function Equals(ByVal obj As Object) As Boolean	Checks if the types have same power for MKSA elements describing dimensions

Classes constructed that implement these methods must be compiled in the form of a *.dll, which must be saved in the R-CRISIS application directory. In addition, the file "CRISISDimensions.ini"[9] must be edited to add the new classes. The general format of the lines of this file is the following:

Full class name, Assembly name

2.4.2 Probabilistic interpretation of attenuation relations

In general, given a magnitude and a distance, intensity A is assumed to be a random variable with a given probability distribution (usually lognormal). GMPE provide the first two probability moments of A given a magnitude and a distance, that is, $A|M,R$. These two moments usually describe the mean or median value of $A|M,R$ and a measure of its uncertainty.

R-CRISIS supports three probability distributions that can be used to describe hazard intensities. These distributions are presented in Table 2-24, together with the two probability moments that have to be given in order to correctly describe $A|M,R$ as a random variable.

[9] Stored at the installation path

Table 2-24 Acceptable probability distributions to describe hazard intensities in R-CRISIS

Distribution	1st moment (μ_1)	2nd moment (μ_2)	Lower limit	A_{max}
Lognormal	Median	Standard deviation of the natural logarithm	0	$\mu_1 \exp(K \mu_2)$
Gamma	Mean	Standard deviation	0	$\mu_1 + K \mu_2$
Normal	Mean	Standard deviation	-infinity	$\mu_1 + K \mu_2$

As part of the hazard computations, R-CRISIS requires to compute the probability that intensity A at a given site exceeds a known value, a, given that at some hypocentral location, H, an earthquake of magnitude M occurred, that is, $\Pr(A>a|M,H)$.

If no truncation is applied to the hazard intensity values, this probability is computed by means of:

$$\Pr(A > a \mid M,H) = 1 - F_A\left[a; \mu_1(M,H), \mu_2(M,H)\right] \quad \text{Eq. (2-45)}$$

where $\mu_1(M, H)$ and $\mu_2(M, H)$ are the first and second probability moments, respectively, of intensity A, given that at hypocentral location H an earthquake of magnitude M occurred. Depending on the probability distribution assigned to A, the first and second probability moments have the interpretation presented in Table 2-24. $F_A[a; \mu_1(M , H),\mu_2(M , H)]$ is the probability distribution of A (also called the cumulative probability function) whose form depends on the type of distribution chosen for the analysis.

The probability moments of $A|M,R$, that is, $\mu_1(M, H)$ and $\mu_2(M, H)$ are provided by the user by means of the GMPE. In many cases, truncation is specified in the GMPE trough a parameter denoted as "*Sigma truncation*", *Tc*. This means that the integration across the attenuation relation uncertainty implied in the previous equations is not carried out up to infinity, but up to a certain value, *Tc*.

Depending on the value of the truncation coefficient given in the GMPE, the following considerations are made:

***Tc*=0**

In this case, no truncation is applied, so equation 2-45 is used.

***Tc*>0**

In this case, a truncated distribution between the lower limit of A and *Tc* is assumed, regardless of magnitude and distance. Hence,

$$\Pr(A>a\,|\,M,H)=\begin{cases}\dfrac{1-F_A\left[a;\mu_1(M,H),\mu_2(M,H)\right]}{1-F_A\left[Tc;\mu_1(M,H),\mu_2(M,H)\right]},a<Tc\\ 0,a>Tc\end{cases}\qquad \text{Eq. (2-46)}$$

Note: when truncating intensities, the original units of the attenuation model should be used regardless any unit factor has been included in the R-CRISIS project.

***Tc*<0**

In this case, ABS(*Tc*)=*K*, is interpreted as the number of standard deviations, for which integration will be performed. Hence, the integration will be performed between the lower limit and A_{max}, both explained in Table 2-24. Therefore,

$$\Pr(A>a\,|\,M,H)=\begin{cases}\dfrac{1-F_A\left[a;\mu_1(M,H),\mu_2(M,H)\right]}{1-F_A\left[A\max;\mu_1(M,H),\mu_2(M,H)\right]},a<A\max\\ 0,a>A\max\end{cases}\qquad \text{Eq. (2-47)}$$

Depending on the distribution chosen, A_{max} takes the values indicated in Table 2-24. Note that in this case, the actual truncation value for *A* depends on magnitude and distance. Figure 2-20 shows the effect of the different truncation schemes.

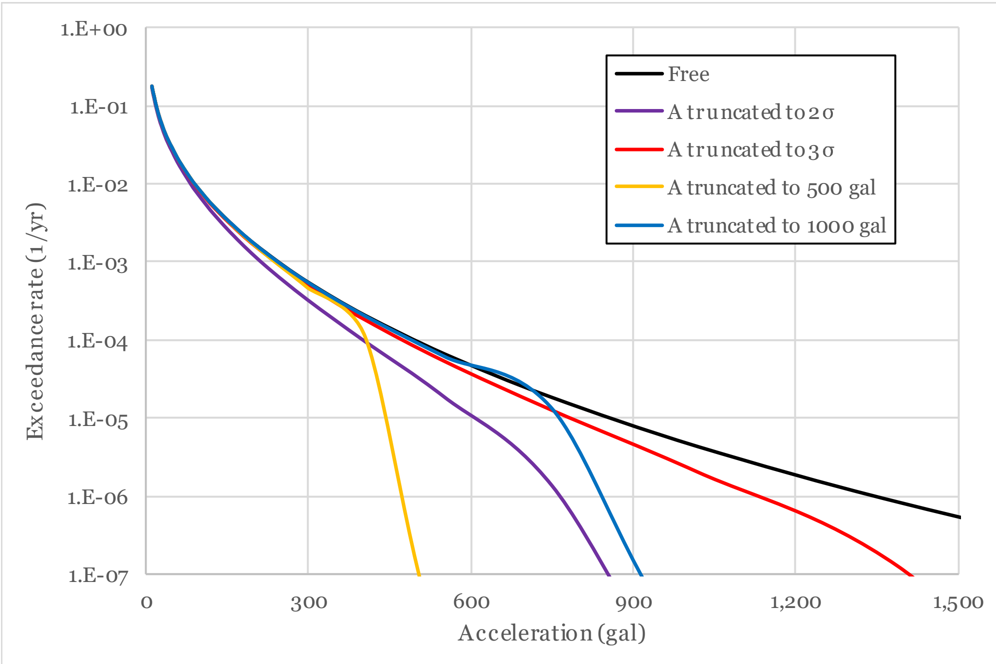

Figure 2-20 Effect of different truncation schemes on GMPM

2.4.3 Built-in GMPEs

As mentioned before, the built-in GMPEs correspond to well-known models published in the literature, that the user can use as attenuation relationships for the R-CRISIS projects. These models, as the user defined attenuation tables, relate in probabilistic terms, earthquake magnitudes and a certain distance measure with the intensity at a computation site. Also, many of these attenuation equations require specification of additional parameters that the user must select, such as style of faulting and soil type.

Table 2-25 includes the list of the available built-in GMPM to date in R-CRISIS and show whereas those have been verified or not. More details about this process are included in Section 4.3 of this document.

The number available built-in models in R-CRISIS expands with time depending on the publication of new models and/or updates of existing ones. Although most of the available built-in GMPEs in R-CRISIS have been included by the developers, users can also provide their inputs through the contact channels available at www.r-crisis.com.

Table 2-25 Built-in GMPEs in R-CRISIS

Reference	Magnitude range	Distance range	Spectral period range
Abrahamson and Silva (1997)	4.0-7.5	0.1-200 km	0.01-5.00 s
Abrahamson et al. (2014) NGA-West2	3.0-8.5	0-300 km	0.0-10.0 s
Abrahamson et al. (2016) BCHydro	5.0-8.4	1-300 km	0.0-3.0 s
Akkar and Bommer (2007)	5.0-7.6	1-100 km	0.0-4.0 s
Akkar and Bommer (2010)	5.0-7.6	1-100 km	0.0-3.0 s
Akkar et al. (2014)	4.0-8.0	0-200 km	0.005-4.0 s
Arroyo et al. (2010)	5.0-8.5	16-400 km	0.001-5.0 s
Atkinson and Boore (2003)	5.0-8.5	1-300 km	0.0-3.0 s
Atkinson and Boore (2006)	3.5-8.0	1-1000 km	0.01-5.0 s
Atkinson (2008)	4.3-7.6	10-1000 km	0.0-5.0 s
Bindi et al. (2011)	4.0-6.9	0.1-200 km	0.0-4.0 s
Bindi et al. (2017)	3.0-8.0	0.1-300 km	0.0-4.0 s
Boore and Atkinson (2008) NGA	5.0-8.0	1-200 km	0.0-10.0 s
Boore et al. (2014) NGA-West2	3.0-8.5	0-400 km	0.01-10.0 s
Campbell (2003)	5.0-8.2	1-1000 km	0.01 - 4.0 s
Campbell and Bozorgnia (2003)	5.0-7.5	1-60 km	0.03-4.0 s
Campbell and Bozorgnia (2008) NGA	4.0-8.5	0-200 km	0.0-10.0 s
Campbell and Bozorgnia (2014) NGA-West2	3.0-8.5	0-300 km	0.0-10.0 s
Cauzzi and Faccioli (2008)	5.0-7.2	6-150 km	0.01-20.0 s
Cauzzi et al. (2015)	4.5-8.0	0-150 km	0.0-10.0 s
Chávez (2006)	4.0-8.5	10-500 km	0.0-5.0 s
Chiou and Youngs (2008) NGA	4.0-8.5	0-200 km	0.0-10.0 s
Chiou and Youngs (2014) NGA-West2	3.5-8.0	0-300 km	0.0-10.0 s
Climent et al. (1994)	4.0-8.0	1-500 km	0.0-5.0 s
Contreras and Boroschek (2012)	5.0-9.0	20-600 km	0.0-2.0 s
Darzi et al. (2019)	4.5-7.4	0-200 km	0.01-10.0 s
Derras et al. (2014)	4.0-7.0	5-200 km	0.0-4.0 s
Derras et al. (2016)	3.5-7.3	3-300 km	0.0-4.0 s
Faccioli et al. (2010)	5.0-7.2	6-150 km	0-20 s
García et al. (2005)	5.0-8.0	0.1-400 km	0.0-5.0 s
Gómez (2017)	3.8-7.1	0.11-634 km	PGA
Idriss (2008)	5.0-8.5	0-200 km	0.01-10.0 s
Idriss (2014) NGA-West2	5.0-8.0	0-150 km	0.01-10.0 s
Jaimes et al. (2006)	5.0-8.4	150-500 km	0.01-6.0 s
Jaimes et al. (2015)	5.2-7.5	103-464 km	0.0-5.0 s
Kanno et al. (2006)	5.5-8.0	1-400 km	0.0-5.0 s
Lanzano et al. (2019)	4.0-8.0	0-200 km	0.04-10.0 s
Lin and Lee (2008)	4.0-8.0	20-250 km	0.0-5.0 s
McVerry et al. (2006)	5.25-8.0	0-400 km	0.0-3.0 s
Montalva et al. (2017)	5.0-9.0	0-300 km	0.01-10.0 s
Pankow and Pechmann (2004)	5.0-7.7	0-100 km	0.01-2.0 s
Pasolini et al. (2008)	4.0-7.0	0-140 km	PGA
Pezeshk and Zandieh (2011)	5.0-8.0	0.1-1000 km	0.0-10.0 s
Pezeshk et al. (2018)	4.0-8.0	0.1-1000 km	0.0-10.0 s
Reyes (1998)	5.0-8.6	150-450 km	0.0-6.0 s
Sabetta and Pugliese (1996)	4.6-6.8	1-100 km	0.1-4.0 s
Sadigh et al. (1997)	4.0-8.0	0.01-200 km	0.0-4.0 s
Sharma et al. (2009)	5.0-7.0	0-100 km	0.0-2.5 s
Spudich et al. (1999) SEA99	5.0-7.5	0.01-100 km	0.0-2.0 s
Tavakoli and Pezeshk (2005)	5.0-8.2	0-1000 km	0.0-4.0 s
Toro et al. (1997)	5.0-8.0	1-500 km	0.0-2.0 s
Yenier and Atkinson (2015)	3.0-8.0	0-600 km	0.0 - 10.0 s
Youngs et al. (1997)	5.0-8.5	10-500 km	0.0-3.0 s
Zhao et al. (2006)	5.0-9.0	0.4-300 km	0.0-5.0 s

Note that in R-CRISIS, besides the parameters that each GMPE uses (e.g. soil type or style of faulting), all built-in GMPEs contain two extra parameters, called "Units coefficient" and

"Sigma truncation". The first one is used to change the original units of the model while the second one is used to truncate the probability distribution of the residuals as explained before.

2.4.4 Generalized GMPE

Generalized attenuation models are non-parametric probabilistic descriptions of the ground motions produced by an earthquake. Ground motions descriptions obtained when using traditional GMPE are generally functions of earthquake magnitude and source-to-site distance as explained in sections 2.4.1 and 2.4.2 but, generalized attenuation models are not explicit functions of magnitude and distance. In the framework of R-CRISIS, a generalized attenuation model is a collection of probabilistic footprints, one for each of the events considered in the analysis. Each footprint provides, in a probabilistic manner, the geographical distribution of the intensities produced by that particular event.

For a given event, the footprint consists of several pairs of grids of values. Each pair of grids is associated to one of the intensity measures for which hazard is being computed. R-CRISIS requires two grids for each intensity measure because, as with other ground motion prediction models, the intensity caused by the earthquake is considered probabilistic and then, to fix a probability density function of the intensity caused by an earthquake at a particular location.

For instance, assume that one generalized attenuation model will be used to describe the intensities caused by 10 different earthquakes. Also, assume that the hazard analysis is being made for seven intensity measures (for instance, the response spectral ordinates for seven different periods). For this example, each event will be described by 14 different grids, two for each intensity measure, the first one providing the geographical distribution of the median intensity and the second one providing the geographical distribution of the standard deviation of the natural logarithm of the intensity. Hence, a total of 140 grids will form the generalized attenuation model of this example. It would be natural that all the 140 grids cover the same geographical extension; however, there are no restrictions at this respect.

From this description, it would be extremely difficult to perform a hazard study of regional (or higher) extension using generalized attenuation models. Usually, a hazard model of regional size contains thousands of events, and the task of geographically describing the intensities caused by each of them in a non-parametric form would be titanic.

Rather, generalized attenuation models are very likely to be used in local studies, for which the relevant earthquakes are few and can be clearly identified. In this case, the grids of required values (geographical distribution of statistical moments of one or more intensity measures for each event) can be constructed using, for instance, advanced ground-motion simulation techniques (Villani et al., 2014).

Generalized attenuation models are provided to R-CRISIS in the form of binary generalized attenuation files (*.gaf extension[10]). The reason for requiring those files to be in binary format

[10] Generalized Attenuation Files

is the computational need of having random access to individual intensity values, something that is basically dictated by computational speed issues.

Table 2-26 shows in detail the format and structure of the *.gaf files.

Table 2-26 Description of the *.gaf file structure

Description	Type	Length	Comments
Custom file description	String	Variable	Provides a brief description of the main features of the GAF
Original units	String	Variable	
Data type (short, integer, single, double, long)	Integer	4	
Probability distribution assigned to intensity (normal, lognormal, beta, gamma)	Integer	4	
Number of intensity measures (e.g. number of fundamental periods)	Integer	4	
Number of sources (locations)	Integer	4	
Number of magnitudes per location	Integer	4	
Number of probability moments of the intensity stored	Integer	4	
Period 1	Double	8	Period values are required since the user may want to compute hazard for arbitrary periods
Period 2	Double	8	
...	...	...	
Period number of intensity measures	Double	8	
Representative magnitude of bin 1	Double	8	Magnitude values are required to compute occurrence rates when G-R or characteristic earthquake models are used. When non-Poissonian seismicity files are used, these magnitudes are irrelevant
Representative magnitude of bin 2			
...	...	...	
Representative magnitude of last bin	Double	8	
Scenario name	Char	40	Magnitude values are required to compute occurrence rates when G-R or characteristic earthquake models are used. When non-Poissonian seismicity files are used, these magnitudes are irrelevant
Grid for intensit measure 1, probability moment 1	ModGRN	56+Nbytes*Nx1*Ny1	
Grid for intensit measure 1, probability moment 2	ModGRN	56+Nbytes*Nx1*Ny1	
...	...	...	

Grid for intensit measure 1, probability moment NumMoments	ModGRN	56+Nbytes*Nx1*Ny1	
Grid for intensit measure 2, probability moment 1	ModGRN	56+Nbytes*Nx1*Ny1	
Grid for intensit measure 2, probability moment 2	ModGRN	56+Nbytes*Nx1*Ny1	
...	...	...	
Grid for intensit measure 2, probability moment NumMoments	ModGRN	56+Nbytes*Nx1*Ny1	
...	...	...	
Grid for intensity measure NumInt, probability moment 1	ModGRN	56+Nbytes*Nx1*Ny1	Then, the actual georeferenced probabilistic intensity values follow
Grid for intensity measure NumInt, probability moment 2	ModGRN	56+Nbytes*Nx1*Ny1	
...	...	...	
Grid for intensity measure NumInt, probability moment NumMoments	ModGRN	56+Nbytes*Nx1*Ny1	
Scenario name	Char	40	Magnitude values are required to compute occurrence rates when G-R or characteristic earthquake models are used. When non-Poissonian seismicity files are used, these magnitudes are irrelevant
Grid for intensit measure 1, probability moment 1	ModGRN	56+Nbytes*Nx2*Ny2	
Grid for intensit measure 1, probability moment 2	ModGRN	56+Nbytes*Nx2*Ny2	
...	...	...	
Grid for intensit measure 1, probability moment NumMoments	ModGRN	56+Nbytes*Nx2*Ny2	
Grid for intensit measure 2, probability moment 1	ModGRN	56+Nbytes*Nx2*Ny2	
Grid for intensit measure 2, probability moment 2	ModGRN	56+Nbytes*Nx2*Ny2	
...	...	...	
Grid for intensit measure 2, probability moment NumMoments	ModGRN	56+Nbytes*Nx2*Ny2	
...	...	...	
Grid for intensity measure NumInt, probability moment 1	ModGRN	56+Nbytes*Nx2*Ny2	
Grid for intensity measure NumInt, probability moment 2	ModGRN	56+Nbytes*Nx2*Ny2	
...	...	...	
Grid for intensity measure NumInt, probability moment NumMoments	ModGRN	56+Nbytes*Nx2*Ny2	
Similar blocks continue for all remaining scenarios			

2.4.5 Hybrid attenuation models

A hybrid (or composite) GMPE is the result of the weighted combination of two or more distributions (usually normal ones) that can have different mean values and standard deviations (Scherbaum et al., 2005). In its most general form, the conditional probability of exceeding an intensity measure A is calculated by means of:

$$P(\mathrm{A} > a) = \sum_{i=1}^{N} \mathrm{w}_i \left\{ 1 - \Phi\left[\frac{a - \mu_i}{\sigma_i} \right] \right\} \quad \text{(Eq. 2-48)}$$

where w_i is the weight assigned to the i^{th} base GMPE, $\Phi[\cdot]$ is the normal distribution and μ_i and σ_i are the mean values and standard deviations respectively of the i^{th} base GMPE. Figure 2-21 shows a schematic representation for the resulting probability function of a hybrid GMPE generated using three base GMPE as well as their weighted probability densities.

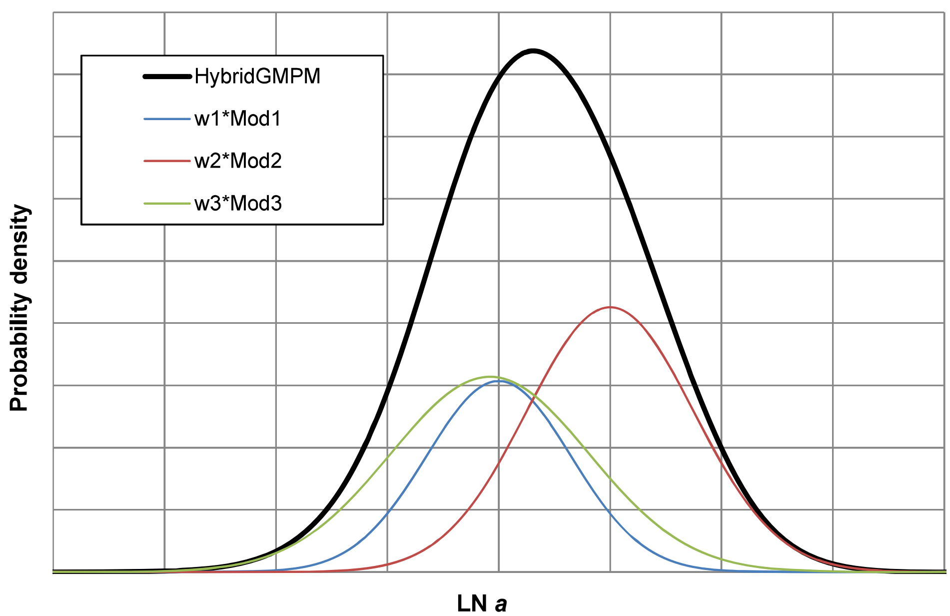

Figure 2-21 Example of a hybrid GMPE

These hybrid GMPE are useful for cases where the normal distributions do not fit well with the recorded earthquake data (i.e. observations show that there are higher probabilities of extremes than those provided by the normal distributions). This issue is more evident, when using normal distributions, at high epsilons and, the development of hybrid GMPE generally allow considering heavier tails as shown in Figure 2-22, which zooms the end tail of Figure 2-21.

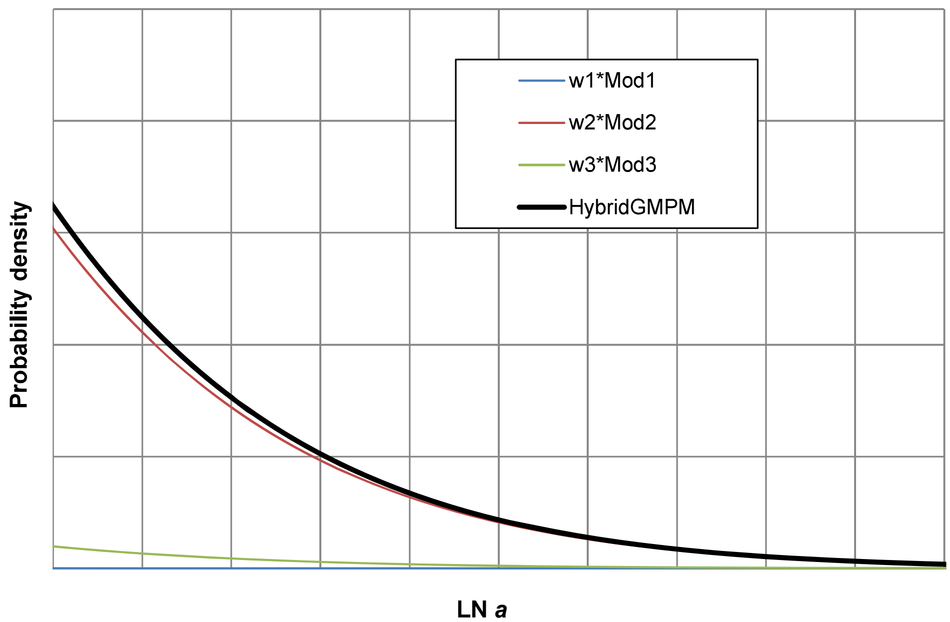

Figure 2-22 Detail of the end tail of the example hybrid GMPE

Note: editing of hybrid GMPEs is restricted in R-CRISIS. In case the user wants to make any change, those must be implemented directly in the base models and, after that, the existing hybrid model must be deleted and created again. Also, care must be taken so that the updated one is properly assigned to the sources in the R-CRISIS project.

<u>Hybrid GMPE vs. logic trees</u>

Hybrid GMPE can be used instead of logic trees when differences in the R-CRISIS models only have to do with the GMPE assignation. Instead of assigning weights to the branches, those are assigned to the base GMPE for the generation of a hybrid attenuation model. Although both approaches produce the same results in terms of expected values because the way in which uncertainties are considered is different (epistemic in the logic trees and random in the hybrid GMPE), the estimations of variances do differ (see Ordaz and Arroyo, 2016).

Note: when hybrid GMPE are used, the seismic hazard intensity is treated as a hybrid random variable and not a lognormal one anymore. Therefore, the second probability moment does not correspond to the standard deviation of the logarithm but to the standard deviation itself.

2.4.6 Special attenuation models

In the most frequent cases, only one attenuation model is assigned to a seismic source. However, there is the possibility to assign one or more special attenuation models to a source, which will be effective only for sites located inside corresponding polygons, called "special attenuation regions" provided by the user. If special attenuation models are given, then R-CRISIS will proceed in the following way:

When computing hazard from a source, R-CRISIS will check if this source has assigned a special attenuation model. If it does not, then it will use the general GMPE assigned to the source. If the source on the other hand has assigned a special attenuation model, then R-CRISIS checks if the site of computation is inside one of the user-provided polygons. If affirmative, R-CRISIS will use the model assigned to this source-site combination. If the site is not inside any of the special polygons, then R-CRISIS will use the general attenuation model assigned to the source.

It must be noted that if site-effects grids are used (see Section 2.5), the amplification factors will be applied on top of the intensities computed either with the general attenuation model assigned to the source or with attenuation models assigned to special attenuation regions. This is of importance to avoid double counting or omission of the site-effects.

2.4.7 Point source (ω^2) attenuation model

R-CRISIS allows developing a GMPE using a point source, ω^2 model based on the following parameters:

- Beta: S-wave velocity in km/s
- C1: first constant required to compute duration
- C2: second constant required to compute duration
- Epsilon
- FFMAX: cut-off frequency, in Hz
- Fmax: maximum frequency for which the GMPM will be calculated
- Fmin: minimum frequency for which the GMPM will be calculated
- FS: free surface amplification factor, usually taken as 2
- t*: near-surface attenuation factor, in s
- Nf: number of frequencies, between Fmin and Fmax for which intensities will be calculated
- NPoles: Number of poles of Butterworth filter
- Q_0: where $Q(f)=Q_0*f^{\varepsilon}$
- Rho: density, in gr/cm^3
- Stress drop: in bar
- Sigma truncation: following the R-CRISIS notation

The units of this GMPE will be always cm/s^2 for accelerations, cm/s for velocities and cm for displacements, although the unit factor field is available. For the case of accelerations, R-CRISIS will automatically estimate the Sa(T) for all the values of the spectral ordinates defined in the seismic hazard project.

Note: the user should review that the frequency range defined for the GMPE covers well enough the spectral ordinates range. Special care must be taken for long period (low frequency) values which can be adjusted through the f_{min} field.

2.5 Site effects

R-CRISIS allows including local site effects in the seismic hazard computations. Site effects are included to the R-CRISIS project in terms of amplification/de-amplification factors that depend on the site location, structural period and ground-motion level (to account for the soil non-linearity).

Amplification factors are interpreted by R-CRISIS in the following way: during the hazard computations, R-CRISIS requires to compute the hazard intensity at structural period T that would take place at site S due to the occurrence of an earthquake of magnitude M originating at distance R. We will denote this intensity as $I(S,T,M,R)$.

Normally, $I(S,T,M,R)$ is computed using the attenuation relationship that the user has selected for the source (either from an attenuation table, a built-in model or a special attenuation model).

The value computed is interpreted by R-CRISIS as the median intensity <u>without</u> site effects but, if site effects data are provided, then the median intensity that R-CRISIS will use for the hazard computations, I_S, is the product of $I(S,T,M,R)$ and the amplification factor defined by the user which as expected, depends on the site location, the structural period and the ground motion level, I_O. This amplification factor is denoted as $A(S,T,I_O)$.

In other words:

$$I_S = (S,T,M,R) = I(S,T,M,R) \cdot A(S,T,I_0) \qquad \text{(Eq. 2-49)}$$

Uncertainty in the hazard intensities after site effects are included can be accounted for in R-CRISIS. If the user has provided not only amplifications factors but also an optional file with the sigma values, the uncertainty measure will be extracted from the latter. If no sigma file has been provided by the user, the standard deviation of the acceleration after site-effects will have the same value than the one it had before site-effects (i.e. that of the GMPE for each spectral ordinate).

The user has to provide R-CRISIS the means to obtain the amplification factors $A(S,T,I_O)$ and, optionally, the uncertainty values $\sigma(S,T,I_O)$. These factors are provided to R-CRISIS by means of two (or three[11]) binary files that are described in the following paragraphs. These files must have the same base name, but different extensions.

Note: if no site-effects are included, $A(S,T,I_O)$=1.0

There are three different ways implemented in R-CRISIS to consider the local site effects and those are denoted as:

- CAPRA Type

[11] If the sigma file is provided

- Chiou and Youngs 2014
- V_{s30}

The complete explanation for each case is presented next from where the structure of the required files can be better understood by the user.

2.5.1 CAPRA Type (ERN.SiteEffects.MallaEfectosSitioSismoRAM)

This approach to consider the local site effects requires providing R-CRISIS a set of files which are used to construct the spectral transfer functions at different locations. The first two are mandatory whereas the third one is optional.

Fundamental period file

This file corresponds to a binary grid file *.grd (in Surfer 6 binary format). The main purpose of this file is to provide a geographical reference for the grid for which the amplification factors are given, as well as to account for the grid's resolution. This grid contains as "z-values" the predominant ground periods associated to each point of the grid. Points with positive periods are interpreted as part of the area for which site effects are known. Points with negative periods are interpreted as outside the area for which site effects are known. Hence, for these points, the amplification factor will always be 1.0 regardless of period and ground motion level. For these points, the uncertainty will be that of the acceleration computed without site-effects.

Extension *.grd is required for this file (e.g. SiteEffects.grd).

Amplification factors file

This file contains the amplification factors themselves. As indicated before, the amplification factors depend on the site location, the structural period and the ground-motion level (if soil non-linearity is considered). In view of this, amplification factors are provided to R-CRISIS by means of a 4-index matrix.

The first two indexes are used to sweep through the geographical extension (i.e. rows and columns of a grid). The size, spacing and extension of the grid containing the amplification factors needs to be the same as for the grid with the predominant periods. The third index sweeps through structural periods, while the fourth index sweeps through ground motion levels.

In principle, amplification factors for a given site and period can be different depending on the size of the ground motion. R-CRISIS uses as an indicator of this size the intensity for the shortest period available for the GMPE that is used to compute the intensity without site effects. It is common practice that for most of the cases (but not always) this intensity corresponds to peak ground acceleration (PGA).

The format in which the amplification factors must be provided to the R-CRISIS project is described in Table 2-27.

Table 2-27 Description of the amplification factors file structure

Block	Variable	Size	Comments
Header	A number 1	Integer	This field is reserved for future use
	Number of ground motion levels, NL	Integer	If NL=1, elastic behavior is assumed
	Number of periods, NT	Integer	
	Ground motion level 1	Double	
	Ground motion level 2	Double	
	...	...	
	Ground motion level NL	Double	
	Period 1	Double	
	Period 2	Double	
	...	...	
	Period NT	Double	
For site 1,1	Amplification function for ground-motion level 1	NT doubles	The amplification function for a give site and ground-motion level is a collection of NT numbers, one for each structural period. The first number is associated to Period 1 and so on
	Amplification function for ground-motion level 2	NT doubles	
	...	...	
	Amplification function for ground-motion level NL	NT doubles	
For site 1,2	Amplification function for ground-motion level 1	NT doubles	The order of the sites is the same as the associated fundamental period grid, starting from the lowest-left cornert and the counter advancing for the columns (i.e. sites are described following the order of cross sections of constant y)
	Amplification function for ground-motion level 2	NT doubles	
	...	...	
	Amplification function for ground-motion level NL	NT doubles	
For site Nx,Ny	Amplification function for ground-motion level 1	NT doubles	Nx and Ny are the number of grid lines along the X axis (columns) and the number of grid lines along the Y axis (rows) provided in the associated period grid file
	Amplification function for ground-motion level 2	NT doubles	
	...	...	
	Amplification function for ground-motion level NL	NT doubles	

The first column of Table 2-28 shows an example of the contents of a site-effects file with extension *.ft; the second column includes some comments about each field.

Table 2-28 Example of site-effects file[12]

Value	Comments
1	A number 1 reserved for future use
3	3 ground motion levels
5	5 different fundamental periods
20	First ground motion level
100	Second ground motion level
300	Third ground motion level
0.0	First period for which amplification factors are provided
0.2	Second period for which amplification factors are provided
0.5	Third period for which amplification factors are provided
1.0	Fourth period for which amplification factors are provided
2.0	Fifth period for which amplification factors are provided
1.3 1.5 2.3 1.0 0.9	Five amplificaton factors, one for each fundamental period for ground-motion level 1
1.2 1.8 2.6 0.9 0.7	Five amplificaton factors, one for each fundamental period for ground-motion level 2
1.1 1.3 2.1 0.6 0.6	Five amplificaton factors, one for each fundamental period for ground-motion level 3
2.3 2.6 3.0 2.2 1.8	Five amplificaton factors, one for each fundamental period for ground-motion level 1
2.2 2.4 3.1 1.9 1.6	Five amplificaton factors, one for each fundamental period for ground-motion level 2
2.1 2.3 3.1 1.7 1.4	Five amplificaton factors, one for each fundamental period for ground-motion level 3
...	...
2.4 2.6 3.4 2.0 1.9	Five amplificaton factors, one for each fundamental period for ground-motion level 1
2.2 2.4 3.1 1.7 1.6	Five amplificaton factors, one for each fundamental period for ground-motion level 2
2.0 2.2 2.9 1.5 1.4	Five amplificaton factors, one for each fundamental period for ground-motion level 3

This data is also provided to R-CRISIS by means of a binary file, with extension *.ft. (e.g. SiteEffects.ft).

Sigma file (optional)

This file contains the values of the uncertainty parameter that will be used instead of that provided by the GMPE if no site-effects are considered. Sigma values depend on the site location, the structural period and the ground-motion level. Dependence on ground-motion level is to account for non-linear soil behavior. In view of this, sigma values are given by means of a 4-index matrix which has the same structure as the matrix than contains the amplification factors (see Table 2-20). If this file is not provided, then the uncertainty after site effects will be the same as uncertainty without site-effects.

This data is also provided through an optional binary file, with extension *.sig. (e.g. SiteEffects.sig).

[12] This file must be in binary format and can be generated using a toolbox included in R-CRISIS

2.5.2 Chiou and Youngs, 2014 (ERN.SiteEffects.MallaVs30CY14)

This approach requires the definition of a fixed V_{s30} value (in m/s) at bedrock level for the area of analysis together with a grid (*.grd format) which contains the variable V_{s30} values, one for each node (again, in m/s). With this data, R-CRISIS calculates the amplification factors using the methodology proposed in the Chiou and Youngs (2014) GMPE.

The soil amplifications, both linear and non-linear, is considered in this case using the proposal by Chiou and Youngs (2014) through an amplification factor, *AF*.

$$AF = \phi_1 \cdot \min\left[\ln\left(\frac{V_{s30}}{1130}\right);0\right] + \phi_2\left[e^{\phi_3(\min(V_{s30};1130)-360)} - e^{\phi_3(1130-360)}\right]\ln\left(\frac{y_{ref}+\phi_4}{\phi_4}\right) \quad \text{(Eq. 2-50)}$$

where ϕ_1, ϕ_2, ϕ_3, ϕ_4 are the coefficients of the site response model provided in Tables 3 and 4 of Chiou and Youngs (2014); Vs_{30} is the travel-time averaged shear-wave velocity (in m/s) at the top 30m of soul and y_{ref} is the ground motion amplitude estimated at bedrock.

The ground motions including the amplification caused by the site effects, y_{se}, are obtained after using the amplification factors on top of the ground motion values obtained from the GMPE (at rock), provided a reference value by the user.

$$y_{se} = y_{ref} \cdot e^{AF} \quad \text{(Eq. 2-51)}$$

where y_{ref} is the ground motion amplitude estimated by the GMPE at bedrock level and *AF* is the Vs_{30}-dependent amplification factor obtained from Equation 2-41.

Units factor

The Chiou and Youngs (2014) *AF* is estimated in terms of g. If the R-CRISIS project uses different units (e.g. cm/s^2), the user must indicate the factor for which the *AF* are to be multiplied for (e.g. if cm/s^2, the unit factor should be equal to 981).

2.5.3 V_{s30} (ERN.SiteEffects.MallaVs30)

This approach requires a grid (*grd format) with the V_{s30} values (in m/s) at different locations. If the selected GMPE used in the R-CRISIS project accounts explicitly for a V_{s30} value in its formulation (e.g. Atkinson and Boore, 2006; Kanno et al., 2006; Atkinson and Boore, 2008; Boore and Atkinson, 2008; Campbell and Bozorgnia, 2008; 2014; Chiou and Youngs, 2008; 2014; Cauzzi and Faccioli, 2008; Idriss, 2008; Abrahamson et al. 2014; 2016) said input value will be read from the site effects grid and therefore, at each computation site a V_{s30} customized GMPE will be used.

2.6 Spatial integration procedure

R-CRISIS assumes that, within a source, seismicity is evenly distributed by unit area for the cases of area and volume sources or by unit length for the cases of line sources. For point and gridded sources, all seismicity is assumed to be concentrated at the points.

In order to correctly account for this modeling assumption, R-CRISIS performs a spatial integration by subdividing the sources originally defined by the user. Once the original source has been subdivided, R-CRISIS assigns to a single point all the seismicity associated to each sub-source, and then the spatial integration adopts a summation form.

The subdivision procedure is briefly described next, although more details about the implemented algorithm are shown in Annex 1.

2.6.1 Area sources

As explained in Section 2.2.1, the geometry of the 3D polygons that represent the seismic sources is described by the user through *N* vertexes for which coordinates (longitude, latitude and depth) are provided. After this, the area source is initially subdivided into *N*-2 triangles. These triangles are further subdivided until one of the following two conditions are met:

1. The size of the triangle is smaller than the value "minimum triangle size" provided to R-CRISIS by the user which means that this is a recursive process where the triangle is subdivided if it is still very big.
2. The ratio between the site-to-source distance and the triangle size is larger than the value "minimum distance/triangle size ratio" provided to R-CRISIS by the user. This is also a recursive process where the triangle is subdivided if the site is still not far enough.

More details about the recursive function used for this purpose are shown in Annex 1. The site-to-source distance is measured from the computation site to the centroid of the triangle whose possible sub-division is being examined. The size of the triangle is simply the square root of its area. At this stage it is worth remembering that the seismicity associated to each centroid is proportional to the triangle's area.

If based on the criterion provided by the user, R-CRISIS decides that a triangle has to be again sub-divided, this process is done by dividing the initial triangle into four new ones, whose vertexes are the mid-points of the three sides of the original triangle.

R-CRISIS uses the following as default parameters:

- Minimum triangle size=11 km.
- Minimum distance/triangle size ratio=3.

Figure 2-23 shows the resulting subdivision of a squared source of size 1°x1° when the computation site is located at the center of the source and using the default integration parameters.

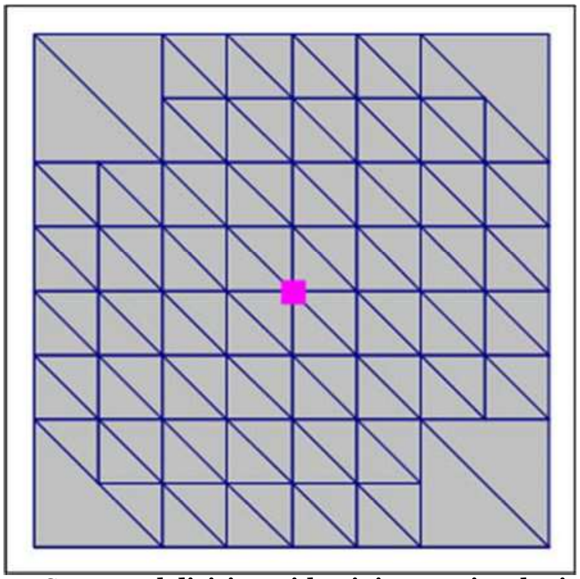

Figure 2-23 Source subdivision with minimum triangle size=11km, minimum distance/triangle size ratio=3

Figure 2-24 shows the same sub-division process but with minimum triangle size=5 km, minimum distance/triangle size ratio=3. Note how, as expected, this sub-division yields smaller triangles in the neighborhood of the computation site.

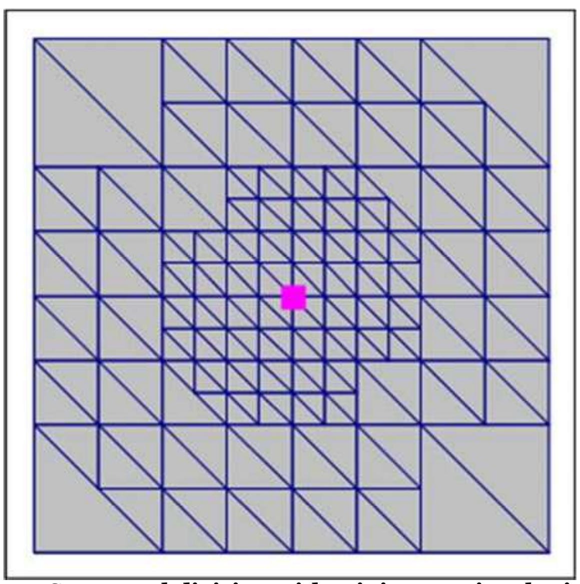

Figure 2-24 Source subdivision with minimum triangle size=5km, minimum distance/triangle size ratio=3

Figure 2-25 shows the same sub-division process but now with minimum triangle size=5 km, minimum distance/triangle size ratio=4. Note that the smaller triangles cover now a wider area around the computation site.

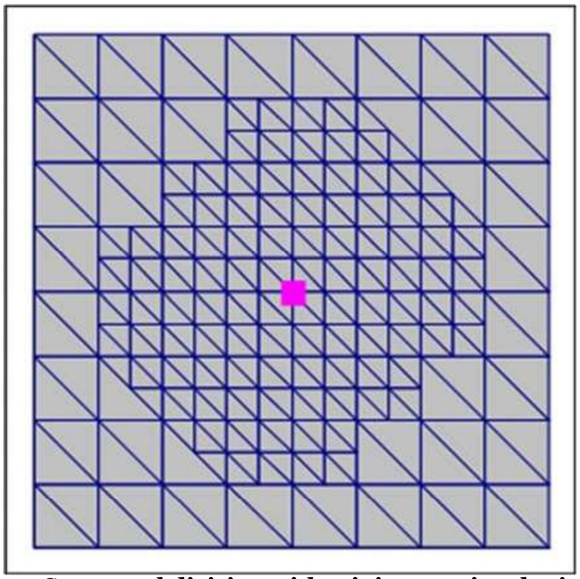

Figure 2-25 Source subdivision with minimum triangle size=11km, minimum distance/triangle size ratio=4

Finally, Figure 2-26 shows the resulting subdivision with minimum triangle size=0.5 km and minimum distance/triangle size ratio=4. Note how the density of triangles varies radially as one move away from the computation site.

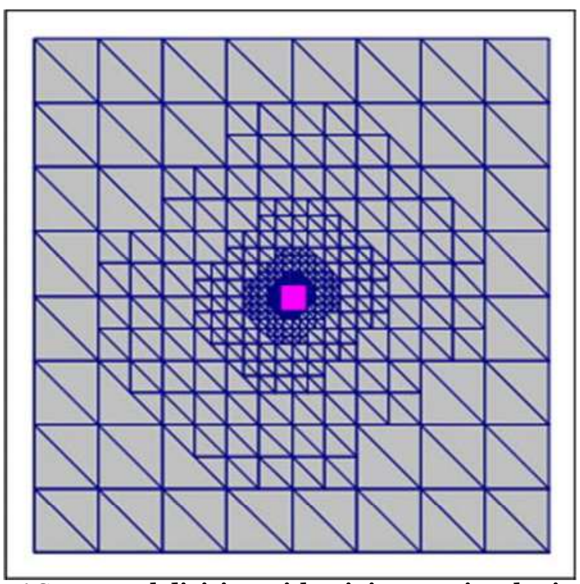

Figure 2-26 Source subdivision with minimum triangle size=0.5km, minimum distance/triangle size ratio=4

2.6.2 Line sources

For this case, the subdivision is performed by the bi-partition of a fault source segment, again until one of the following criteria are met:

1. The size of the line is smaller than the value “minimum triangle size” defined by the user.
2. The ratio between the site-to-source distance and the line size is larger than the value “minimum distance/triangle size ratio” defined by the user.

The site-to-source distance is measured from the computation site to the midpoint of the line whose possible subdivision is being examined. The size of the line corresponds simply to its length. In this case, the seismicity associated to each centroid is proportional to the line’s length.

2.7 Use of a digital elevation model (DEM)

R-CRISIS allows including of a digital elevation model (DEM) to be used in the seismic hazard computations. The DEM is provided to R-CRISIS in terms of elevation values (in km) for each location.

The elevation values are interpreted by R-CRISIS in the following way: during the hazard computations, R-CRISIS requires to compute the ground motion intensity due to an earthquake of magnitude M, with focal depth H, at the distance R between the source from which it was originated and the computation site.

Originally, the distance and depth are estimated assuming that the computation site is located at altitude 0. However, if the user includes a DEM, the altitude of the computation site will be that given by the DEM, which will have an influence on the computation of both, distance and focal depth.

Figure 2-27 illustrates the way in which R-CRISIS calculates distance and depth when a DEM is provided whereas Figure 2-28 shows a top view in which the differences between R_{RUP} and R_{JB} can be better understood. For a practical case study on the use of this feature, see more details in Peruzza et al. (2017).

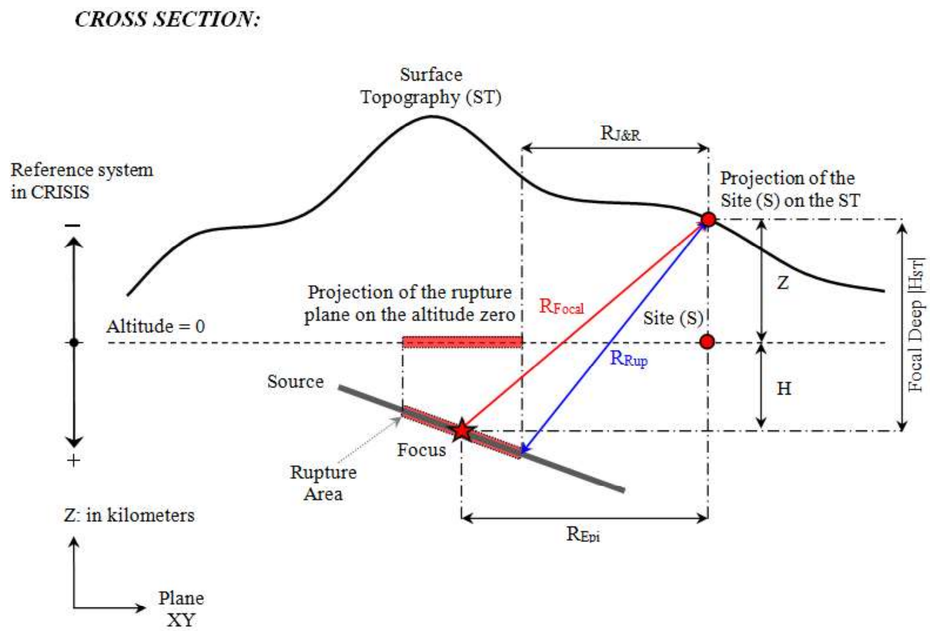

Figure 2-27 Measurement of distances when using DEM

where:

- Z: Elevation value at the computation site
- H: Focal depth relative to zero altitude
- H_{ST}: Focal depth measured from the surface topography to the hypocenter
- R_F: Focal distance
- R_{EPI}: Epicentral distance
- R_{JB}: Joyner and Boore distance (closest distance to the projection of the fault plane at altitude zero)
- R_{RUP}: Closest distance to rupture area

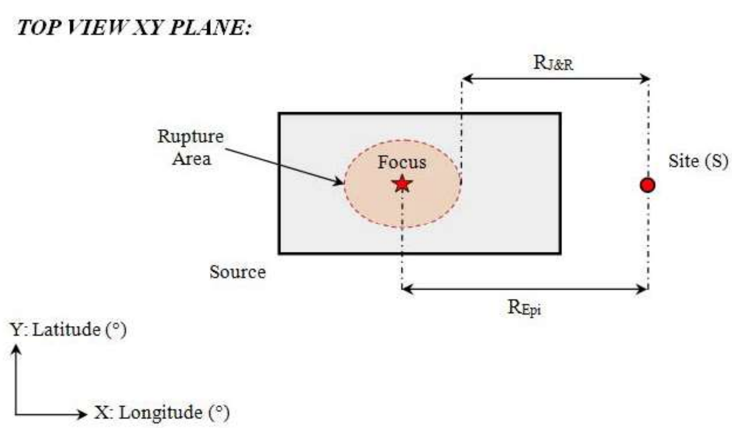

Figure 2-28 Top view of R_{JB} and R_{RUP} distances

In R-CRISIS, the DEM is provided by means of a Surfer grid file[13] (either in Surfer 6 binary or Surfer 6 ASCII formats). The main purpose of this file is to locate in space the grid of altitude values, as well as to provide the grid's spatial resolution. This grid contains as z-values the ground altitude, in km, associated to each point of the grid. Points with positive values are interpreted as above sea level and points with negative values as sites below sea level.

Figure 2-29 shows schematically the difference of considering or not a DEM at a city located at high altitude with respect the mean sea level (e.g. Mexico City, Bogotá D.C., La Paz). It is evident that in the case where the DEM has been considered, since computation distances are larger, exceedance probabilities, mainly for higher intensities are lower; although this of course depends highly on the GMPE used in the PSHA.

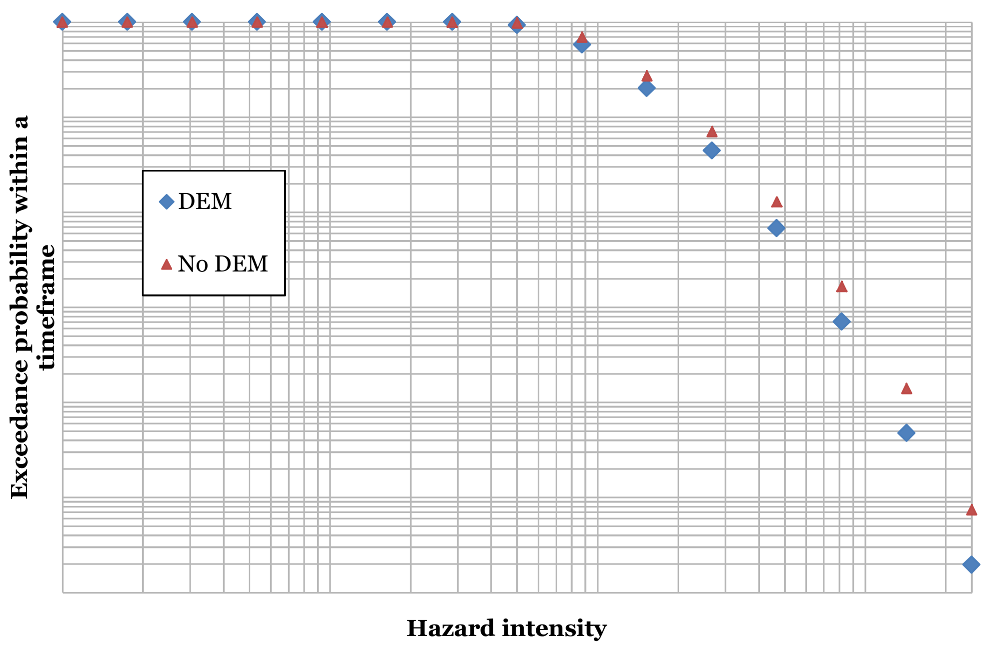

Figure 2-29 DEM v.s. no DEM seismic hazard results

2.8 Combination of seismicity, geometric and attenuation models

Different geometry, seismicity and attenuation models can be combined in R-CRISIS and this section shows which of those combinations are feasible to be used. Tables 2-24 and 2-25 show the validity of the combinations for different seismicity, geometric and attenuation models. In all of them, the color codes indicate the following:

- Green: Combination that is always valid regardless of the parameters values
- Yellow: Combination that is valid, or not, depending on the parameters values
- Red: Combination that is never valid
- Blue: Combination that is potentially valid but not yet implemented

[13] *.grd extension

2.8.1 Normal attenuation models

Table 2-29 shows the validity of the combinations for normal attenuation models (i.e. attenuation tables and built-in GMPM).

Table 2-29 Feasibility of normal attenuation, geometric and seismicity models combination

Geometric model / Seismicity model	Modified G-R	Characteristic earthquake	Generalized non-Poissonian	Generalized Poissonian	Gridded seismicity
Area	A	A	B	B	E
Line	A	A	B	B	E
SSG	C	E	D	D	E
Area-planes	A	A	B	B	E
Grid	E	E	E	E	E

The codes on each field mean the following:

- A: These are options available since previous CRISIS versions that are always valid.
- B: In this option a source is represented by means of line or area geometry model which means that every point that belongs to the source has the same probability of being a hypocenter (the usual assumption when using these geometry models in R-CRISIS). Attenuation models, as in previous versions of CRISIS are constructed using a parametric description (normal GMPE). Anyhow, the new option allows considering the earthquake occurrence probabilities with a generalized Poissonian or non-Poissonian model and not by means of parametric frequency-magnitude relations (i.e. G-R or characteristic earthquake). The occurrence probabilities provided in the Poissonian or non-Poissonian seismicity files correspond to the whole seismic source, that is to be understood as having the probabilities of earthquakes of given magnitudes and within a timeframe anywhere within the source. Using the spatial integration algorithm, explained in Section 2.6, R-CRISIS will sample the source in order to compute hazard accounting for all possible locations of the earthquakes inside it. Not that however, when probabilities are specified for the whole source, those associated to segments of it or to the sub-sources are not univocally defined. The following approach is adopted by R-CRISIS in order to define the occurrence probabilities associated to sub-sources with known sizes.

  Assuming that there is a conventional Poissonian source, the probability of having i events of magnitude M in the next Tf years and accounting for the participation of the whole fault, $P(i,M,Tf)$, is given by:

  $$P(i,M,T_f) = \exp(-\Delta\lambda(M)T_f) \quad \text{(Eq. 2-52)}$$

  where $\Delta\lambda(M)$ is the Poissonian magnitude occurrence rate of earthquakes with magnitudes in the vicinity of M, again for the whole source. This occurrence rate can be written as:

$$\Delta\lambda(M) = -\text{Ln}\left[\frac{P(i,M,T_f)}{T_f}\right] \quad \text{(Eq. 2-53)}$$

In the case of Poissonian occurrences, it is well known that rates are additive and thus, the occurrence rate corresponding to a sub-source of relative size w_j is:

$$\Delta\lambda_j(M) = \Delta\lambda(M)\cdot w_j \quad \text{(Eq. 2-54)}$$

When considering all sub-sources, it is evident that Σw_j=1.0. Knowing this, the occurrence probability associated to the sub-source *j* is:

$$P_j(i,M,T_f) = \exp(-\Delta\lambda_j(M)T_f) = \exp(-\Delta\lambda(M)T_f\cdot w_j) = \exp(\text{Ln}\left[P(i,M,T_f)\right]w_j) \quad \text{(Eq. 2-55)}$$

From which it evident that:

$$P_j(i,M,T_f) = P_j(i,M,T_f)^{w_j} \quad \text{(Eq. 2-56)}$$

If only the occurrence probabilities for the whole source are specified, there is not a unique way to define the occurrence probabilities associated to the sub-sources. Anyhow, the approach followed by R-CRISIS is very reasonable, besides being exact for the case of the Poissonian sources.

The only compatibility restriction when using this option is that the file that contains the non-Poissonian occurrence probabilities must include (in the *.nps file) that the number of sources is equal to 1, which means that only a set of occurrence probabilities is provided. See section 2.1.4 to see where this parameter is to be included.

Note: within the CRISIS development team, this combination is known as *Peruzza type* since Prof. Laura Peruzza suggested its implementation and used it during the calculations made in the context of Project S2 (2008-2010) funded by the Italian Civil Protection Authority (Italian Research Project INGV-DPC S2).

- C: For this option, the point geometry model is used together with a normal attenuation model and a parametric seismicity description (either modified G-R or characteristic earthquake). This is an option available in previous versions of CRISIS and there are not compatibility restrictions.

- D: In this option, the point geometry model is used together with normal attenuation models and earthquake probabilities defined by means of generalized Poissonian and non-Poissonian models. This option is mainly used to model the so-called smoothed seismicity but now with probabilities obtained with spatially arbitrarily complex Poissonian or non-Poissonian models. The only compatibility restriction in this option is that the number of vertexes used in the description of the point-sources must be equal to the number of sources provided in the Poissonian or non-Poissonian seismicity files.

Note: within the CRISIS development team, this combination is known as *Warner-type* since Dr. Warner Marzocchi suggested its implementation and used it during the calculations made in the context of Project S2 (2008-2010) funded by the Italian Civil Protection Authority (Italian Research Project INGV-DPC S2).

- E: The gridded seismicity model only works currently together with grid sources are used as geometry model.

2.8.2 Generalized attenuation models

Table 2-30 shows the validity of the combinations for generalized attenuation models.

Table 2-30 Feasibility of generalized attenuation, geometric and seismicity models combination

Geometric model / Seismicity model	Modified G-R	Characteristic earthquake	Generalized non-Poissonian	Generalized Poissonian	Gridded seismicity
Area	AG	AG	BG	BG	EG
Line	AG	AG	BG	BG	EG
SSG	CG	CG	DG	DG	EG
Area-planes	AG	AG	BG	BG	EG
Grid	CG	CG	DG	DG	FG

The codes on each field in this case mean the following:

- AG: In this option, line or area geometry models are used and ground motion characteristics are described by means of a generalized attenuation model (see section 2.4.3). This option is not possible to use since generalized attenuation models are associated to known, fixed focal locations while line or area sources account, implicitly for uncertainty about the location of future hypocentres being then incompatible.

  In addition, generalized attenuation models contain information about individual events with known (although in some cases irrelevant) magnitudes. Since each event is associated to a fixed value of magnitude, occurrence probabilities for each of the events included in the attenuation model cannot be computed for continuous, arbitrary values of magnitude with the information provided by parametric seismicity descriptions, such as earthquake magnitude exceedance rates. It is important to remember that, starting with magnitude exceedance rates, occurrence probabilities within given timeframes can only be computed for magnitude intervals (magnitude "bins") and not for point values.

- BG: In this option, line or area geometry models are used, seismicity is described by means of a generalized non-Poissonian model and ground motion characteristics are provided through a generalized attenuation model. This is the only option in which generalized attenuation models can be used.

  Note that when using this type of ground motion model, locations of earthquake hypocenters are, in principle, unknown and irrelevant. In consequence, specification of a source location is also, in principle, irrelevant. However, there are two reasons that justify why a source location must by specified:

1. When developing a hazard model using the R-CRISIS interface, it is useful for the modeler to have a visual reference of the source location and,
2. For hazard disaggregation purposes (see Section 2.10), R-CRISIS must know the location to which the hazard coming from all events has to be assigned. For hazard disaggregation purposes, earthquake location is conventionally considered to be the geometrical center of the line or the area source.

On the other hand, since also earthquake magnitudes are fixed (and again, irrelevant) in generalized attenuation models, and each set of grids that represent individual events, it would be impossible to associate the seismicity parameters of the events using parametric descriptions. In view of this, the only possibility is that earthquake occurrence probabilities are assigned using non-Poissonian generalized models. The compatibility conditions for the use of this option are the following:

1. The number of sources in the generalized attenuation model file (*.gaf) must be the same that the number of sites in the generalized non-Poissonian seismicity file (*.nps).
2. The number of magnitudes in the generalized attenuation model file (*.gaf) must be the same that the number of sites in the generalized non-Poissonian seismicity file (*.nps).

Note: within the CRISIS development team, this combination is known as *Stupazzini-Villani type* since Marco Stupazzini and Manuela Villani were the two researches in charge of its development in the context of Project S2 (2008-2010) funded by the Italian Civil Protection Authority (Italian Research Project INGV-DPC S2).

- CG: In this option the geometry of the sources is described through a collection of points and ground motion characteristics using a generalized attenuation model. The use of this combination is considered as impossible since, generalized attenuation models, contain information about individual events with known (although irrelevant) magnitudes. Since each event is associated to a fixed value of magnitude, occurrence probabilities for each of the events contained in the attenuation model cannot be computed for continuous and arbitrary values of magnitude with the information provided by parametric seismicity descriptions (e.g. earthquake magnitude exceedance rates). It is important to remember that, starting with magnitude exceedance rates, occurrence probabilities within given timeframes can only be computed for magnitude intervals (magnitude “bins”) and not for point values.

- DG: Note that this option is like BG except that the source geometry in this case is of point-source type. In principle, this option could have been considered as valid since, when using generalized attenuation models, source geometry is irrelevant. However, the BG option (in which sources can be seen by the modeler) is considered more useful and this one has been inhibited in R-CRISIS to avoid any possible confusion.

- EG: Although methodologically possible, this combination has not yet been implemented.

2.9 Hazard computation algorithm

To compute seismic hazard, the territory under study is first divided into seismic sources according to geotectonic considerations (Cornell, 1968; Esteva, 1970). In most cases, it is assumed that, within a seismic source, an independent earthquake-occurrence process is taking place. For each seismic source, earthquake occurrence probabilities are estimated by means of statistical analysis of earthquake catalogues.

In the more general case, earthquake occurrence probabilities must stipulate the probability of having *s* events (*s*=0, 1, ..., *Ns*) of magnitude M_i in the following T_j years at a given source *k*. We will denote these probabilities as $P_k(s,M_i,T_j)$ and they completely characterize the seismicity of source *k*.

Seismic hazard produced by an earthquake of magnitude M_i at a single point source, say the k^{th} source and for the next T_j years, can be computed as:

$$\Pr(A \geq a \mid M_i, T_j, k) = 1 - \sum_{s=0}^{Ns} P_k(s, M_i, T_j)\left[1 - \Pr(A \geq a \mid M_i, R_k)\right]^s \qquad \text{(Eq. 2-57)}$$

where $Pr(A \geq a|M_i,R_k)$ is the probability that intensity *a* is exceeded given that an earthquake of magnitude M_i occurred at source *k*, that is separated from the site of interest by a distance R_k. Please note that this probability depends only on magnitude, *M*, and source-to-site distance, *R*, and it is normally computed using the probabilistic interpretation of intensities through the use of GMPM. We also note that implicit in equation 2-46 is the assumption that exceedances of intensity values at source *k*, given that an earthquake of magnitude M_i occurred, are independent from each other. This is the reason why the non-exceedance probability of a given *s* events of magnitude M_i occurred at source *k* can be computed as $[1-Pr(A \geq a|M_i,R_k)]^s$.

Seismic hazard, contained in equation 2-57, is more easily expressed in terms of non-exceedance probabilities in the following manner:

$$\Pr(A \leq a \mid M_i, T_j, k) = \sum_{s=0}^{Ns} P_k(s, M_i, T_j)\left[\Pr(A \leq a \mid M_i, R_k)\right]^s \qquad \text{(Eq. 2-58)}$$

Equation 2-58 gives the non-exceedance probability of intensity value *a* given that only earthquakes of magnitude M_i occurred. The non-exceedance probability of *a*, associated to the occurrence of earthquakes of all magnitudes at source *k* in the next T_j years can be computed as:

$$\Pr(A \leq a \mid T_j, k) = \prod_{i=1}^{Nm} \Pr(A \leq a \mid M_i, T_j, k) \qquad \text{(Eq. 2-59)}$$

where Nm is the number of magnitude bins into which the earthquake occurrence process has been discretized. Again, we have used the independence hypothesis among earthquakes of all magnitudes.

But seismic sources are usually points, lines, areas or volumes, so a spatial integration process must be carried out to account for all possible focal locations. We will assume that the spatial integration process leads to N sources. So finally, if earthquake occurrences at different sources are independent from each other, we obtain that the non-exceedance probability of intensity a in the next T_j years due to earthquakes of all magnitudes located at all sources, can be computed with:

$$\Pr(A \le a \mid T_j) = \prod_{k=1}^{N} \Pr(A \le a \mid T_j, k) \qquad \text{(Eq. 2-60)}$$

$$\Pr(A \le a \mid T_j) = \prod_{k=1}^{N} \prod_{i=1}^{Nm} \Pr(A \le a \mid M_i, T_j, k) \qquad \text{(Eq. 2-61)}$$

$$\Pr(A \le a \mid T_j) = \prod_{k=1}^{N} \prod_{i=1}^{Nm} \sum_{s=0}^{Ns} P_k(s, M_i, T_j) \left[\Pr(A \le a \mid M_i, R_k)\right]^s \qquad \text{(Eq. 2-62)}$$

Finally,

$$\Pr(A > a \mid T_j) = 1 - \prod_{k=1}^{N} \prod_{i=1}^{Nm} \sum_{s=0}^{Ns} P_k(s, M_i, T_j) \left[\Pr(A \le a \mid M_i, R_k)\right]^s \qquad \text{(Eq. 2-63)}$$

Equation 2-63 is the one used by R-CRISIS to compute seismic hazard for situations in which the sources are spatially distributed (k=1,…,N), there are earthquakes of various magnitudes (M_i, i=1,…Nm) and the earthquake occurrence probabilities in known time frames T_j at source k are defined by $P_k(s,M_i,T_j)$, that is, the probability of having s events of magnitude M_i in the next T_j years occurring at source k.

The equations presented herein are, in general, applicable to non-Poissonian occurrence processes. But they are also applicable to the Poissonian process. Let us see what results we obtain if we assume that the occurrence process is Poissonian. Let us assume that in all sources, a Poissonian occurrence process is taking place for earthquakes of all magnitudes. Under this assumption, $P_k(s,M_i,T_j)$ takes the form of, precisely, a Poisson probability distribution:

$$P_k(s, M_i, T_j) = \frac{\left[\Delta\lambda_k(M_i) T_j\right]^s \exp\left[-\Delta\lambda_k(M_i) T_j\right]}{s!}, s \ge 0 \qquad \text{(Eq. 2-64)}$$

where $\Delta\lambda_k(M_i)$ is the number of earthquakes of magnitude M_i that, per unit time, take place at source k. In other words, this quantity is the conventional exceedance rate of earthquakes in the range of magnitudes represented by M_i, that is,

$$\Delta\lambda_k(M_i) = \lambda_k\left(\frac{M_i - \Delta M}{2}\right) - \lambda_k\left(\frac{M_i + \Delta M}{2}\right) \tag{Eq. 2-65}$$

Replacing equation 2-55 in equation 2-49 we obtain:

$$\Pr(A \le a \mid M_i, T_j, k) = \sum_{s=0}^{\infty} \frac{\left[\Delta\lambda_k(M_i)T_j\right]^s \exp\left[-\Delta\lambda_k(M_i)T_j\right]}{s!}\left[\Pr(A \le a \mid M_i, R_k)\right]^s \tag{Eq. 2-66}$$

Note that now the sum extends to infinity since, in the Poisson process, the possible range of values of s ranges from zero (0.0) to infinity. The sum in equation 2-57 has an analytical solution:

$$\Pr(A \le a \mid M_i, T_j, k) = \exp\left\{-\Delta\lambda_k(M_i)T_j\left[1 - \Pr(A \le a \mid M_i, R_k)\right]\right\} \tag{Eq. 2-67}$$

$$\Pr(A \le a \mid M_i, T_j, k) = \exp\left\{-\Delta\lambda_k(M_i)T_j \Pr(A \ge a \mid M_i, R_k)\right\} \tag{Eq. 2-68}$$

Hence, from equation 2-63 we get that

$$\Pr(A > a \mid T_j) = 1 - \prod_{k=1}^{N}\prod_{i=1}^{Nm} \exp\left\{-\Delta\lambda_k(M_i)T_j \Pr(A \ge a \mid M_i, R_k)\right\} \tag{Eq. 2-69}$$

$$\Pr(A > a \mid T_j) = 1 - \exp\left\{-\sum_{k=1}^{N}\sum_{i=1}^{Nm} \Delta\lambda_k(M_i)T_j \Pr(A \ge a \mid M_i, R_k)\right\} \tag{Eq. 2-70}$$

But, under the Poissonian assumption for the earthquake occurrences, the process of intensity exceedances follows also a Poissonian process, for which the exceedance probability of intensity a during the next T_j years is given by:

$$\Pr(A > a \mid T_j) = 1 - \exp\left\{-\upsilon(a)T_j\right\} \tag{Eq. 2-71}$$

where $\nu(a)$ is the exceedance rate of intensity a. Comparing equations 2-70 and 2-71 we obtain that

$$\nu(a) = \sum_{k=1}^{N}\sum_{i=1}^{Nm} \Delta\lambda_k(M_i)\Pr(A \ge a \mid M_i, R_k) \tag{Eq. 2-72}$$

Note that *ν(a)*, the well-known Poissonian intensity exceedance rate, does not depend anymore on T_j. In the limit, the inner sum of equation 2-61 can readily be recognized as the integral with respect to magnitude that is present in the conventional Esteva-Cornell approach (Cornell, 1968; Esteva, 1970) to compute Poissonian seismic hazard. The outer sum in equation 2-72 is simply the aggregation of intensity exceedance rates due to all sources. In other words:

$$\nu(a) = \sum_{k=1}^{N} \sum_{i=1}^{Nm} \frac{\Delta\lambda_k(M_i)}{\Delta M} \Pr(A \geq a \mid M_i, R_k)\Delta M \qquad \text{(Eq. 2-73)}$$

$$\nu(a) = \sum_{k=1}^{N} \int_M -\frac{d\lambda_k(M)}{dM} \Pr(A \geq a \mid M, R_k) dM \qquad \text{(Eq. 2-74)}$$

Note that, due to the definition we used for $\Delta\lambda_k(M_i)$ in equation 2-73, its sign changed when we converted it to its differential form. We have then shown that equation 2-63, derived for the general non-Poissonian case, is also valid for the Poissonian case, leading to the well-known Esteva-Cornell expression to compute seismic hazard.

The maximum integration distance is a value provided by the user to the R-CRISIS project and also, the way it is spaced between the lower and upper limits of the hazard intensities for each spectral ordinate can be defined. This last refers to the number of points for which the hazard curve is constructed as well as its spacing. Linear and logarithmic scales can be selected. Figure 2-30 schematically shows the results for the same computation site in terms of annual exceedance probabilities with hazard curves constructed by 5 and 15 points, respectively.

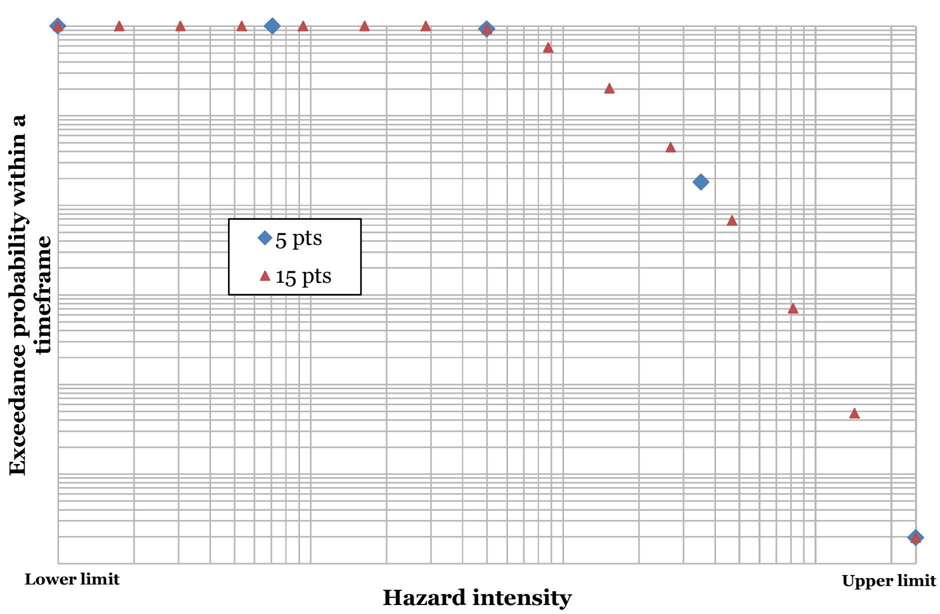

Figure 2-30 Differences due to the number of intensity levels in the hazard plot

Figure 2-31 shows the difference when again, for the same calculation site and using 15 intensity levels, linear and logarithm spacing scales are used.

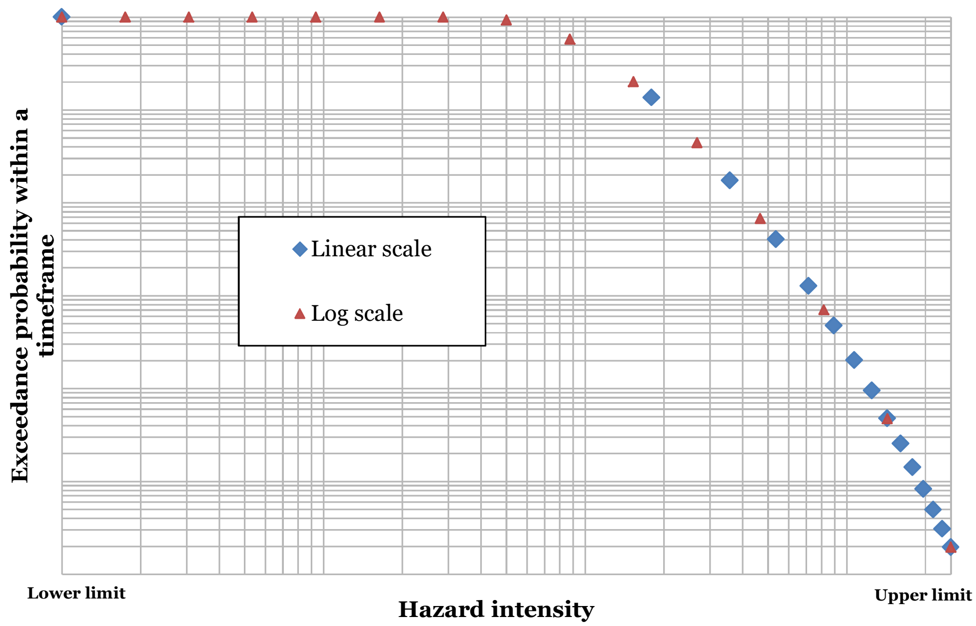

Figure 2-31 Differences due to the distance scaling in the hazard plot

<u>Note:</u> starting with CRISIS2008, the code does not work anymore with intensity exceedance rates as measures of seismic hazard. The more recent versions estimate seismic hazard in terms of probabilities of exceedance of intensity values in given time frames. For instance, a valid measure of seismic hazard in the newer versions is the probability of experiencing peak ground acceleration greater or equal than 0.20g in the next 50 years at a given location. This change was made in order to allow users to introduce in the computations probabilities of earthquake occurrences derived from non-Poissonian models. Poissonian computations, however, are still possible since one can regard this case as a particular case of the non-Poisson computations.

2.10 Hazard disaggregation

2.10.1 Magnitude-distance disaggregation

Consider the basic hazard computation equation (same as equation 2-61 but repeated herein for convenience of the reader)

$$\Pr(A \le a \mid T_j) = \prod_{k=1}^{N} \prod_{i=1}^{Nm} \Pr(A \le a \mid M_i, T_j, k) \qquad \text{Eq. (2-61*)}$$

where $\Pr(A<a|T_j)$ is the probability of not exceeding intensity a at a site in the next T_j years, when subjected to a seismic regime composed by N point sources, each of which produces earthquakes of magnitudes $M_1, M_2, \ldots, M_{Nm}$. It can be noted that the product in equation 2-

61* is composed by many terms, each of which corresponds to a particular magnitude value, M_i, and to a specific source-to-site distance, which is the one from source k to the site for which hazard is being computed.

In view of this, the contributions to Pr($A<a|T_j$) or to Pr($A>a|T_j$) could be grouped for a range of magnitudes (i.e. from M_1 to M_2) and a range of distances. This is the magnitude-distance disaggregation. These results indicate which combinations of magnitude and distance contribute more to the seismic hazard at a site, for a given intensity measure, for a given time frame and at certain level of intensity, a in this case.

Let's say that hazard has been disaggregated, leading to a matrix of Ng rows (one for each magnitude range) and Nr columns (one for each distance range). The contents of each cell must be such that the following relation is satisfied:

$$\Pr(A \le a \mid T_j) = \prod_{l=1}^{N_r} \prod_{m=1}^{Nm} p_{lm} \qquad \text{Eq. (2-75)}$$

In other words, the original non-exceedance probability must be equal to the product of the non-exceedance probabilities disaggregated for each magnitude-distance bin. This means that, opposite to what happens with intensity exceedance rates, which are additive, non-exceedance probabilities (or exceedance probabilities) are not additive but multiplicative, in the sense expressed by equation 2-75. In view of this, when interpreting R-CRISIS disaggregation results, the user must not expect that the exceedance probabilities associated to each cell used for the disaggregation add up to the total exceedance probability computed for the same site, intensity value and time frame.

Note: arithmetic of exceedance probabilities is more complex to that of intensity exceedance rates used in conventional hazard studies.

2.10.2 Epsilon disaggregation

In occasions, it is interesting to know which portions of the intensity probability density function contribute most to the seismic hazard at a given site. Consider the following equation, which is equation 2-61* but written in terms of exceedance probabilities:

$$\Pr(A > a \mid T_j) = 1 - \prod_{k=1}^{N} \prod_{i=1}^{Nm} \left[1 - \Pr(A > a \mid M_i, T_j, k) \right] \qquad \text{Eq. (2-76)}$$

For a given magnitude, time frame and source location, the term Pr($A>a|Mi,Tj,k$) will be computed by calculating the area shown in green in Figure 2-32.

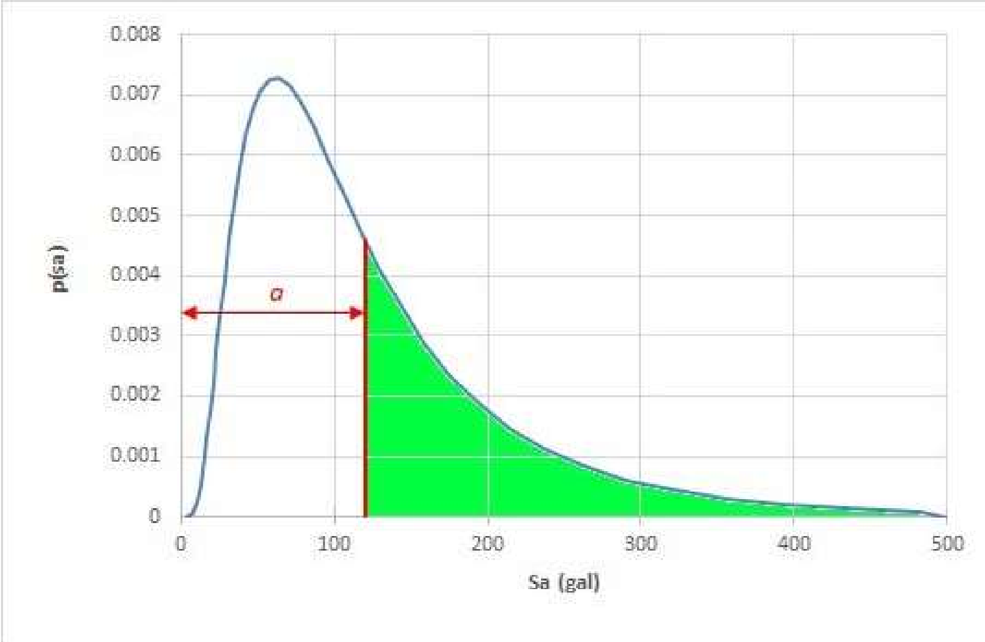

Figure 2-32 Estimation of the non-exceedance probability for given median and standard deviation of the natural logarithm

The example in Figure 2-32 corresponds to a case in which acceleration has a lognormal distribution with median, *MED*(*A*|*M1,Tj,k*) equal to 120 cm/s² and standard deviation of the natural logarithm, σ_{LN}, equal to 0.7.

The shape of the probability density function of *Sa* depends on magnitude, distance, and GMPM employed, while *a* is an arbitrarily fixed value: the one for which seismic hazard is being computed.

However, it is sometimes of interest to know how much of the probability marked in green in Figure 2-32 comes from the high percentiles of the distribution. For instance, how much of the green probability comes from the area to the left of value A_{eps} shown in orange in Figure 2-33.

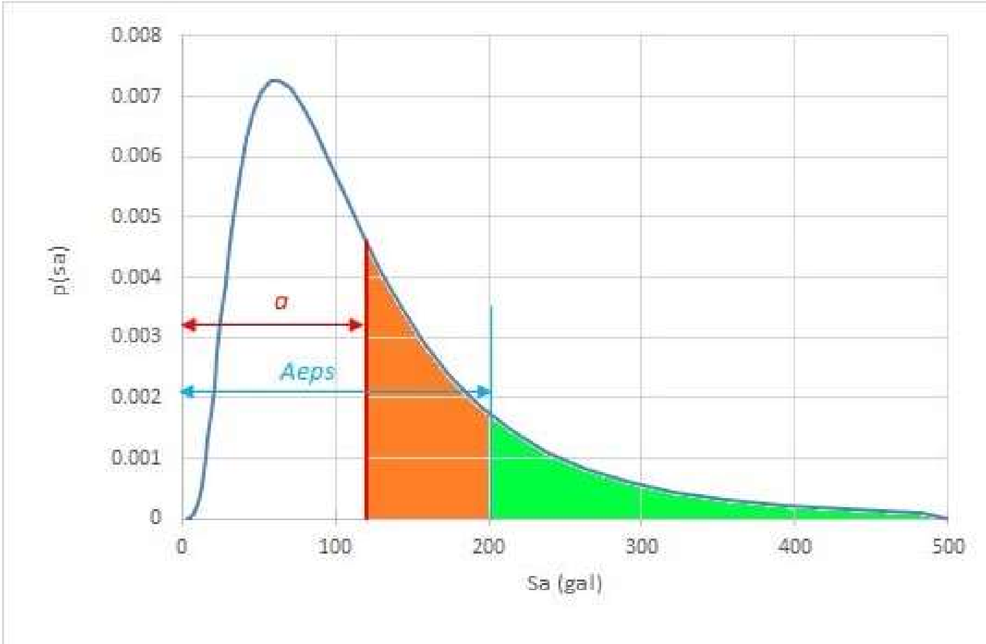

Figure 2-33 Estimation of A_{eps}

Normally, A_{eps} is indexed to an "epsilon" (ε) value, such that:

$$A_{eps} = MED(A \mid M_i, T_j, k)\exp\left[\varepsilon \sigma_{LN}(A \mid M_i, T_j, k)\right] \qquad \text{Eq. (2-77)}$$

where $MED(A|Mi, Tj, k)$ and $\sigma_{LN}(A|Mi, Tj, k)$ are, respectively, the median and the logarithmic standard deviation of A given the occurrence of an earthquake with magnitude Mi at source k; the value of ε is kept fixed for the whole analysis. In the case of Figure 2-29, ε=2 and therefore, A_{eps}=120*exp(2*0.7)=201.37. In view of this, when an epsilon disaggregation is required, exceedance probabilities required to evaluate equation 2-77 are computed with:

$$\Pr(A > a \mid M_i, T_j, k) = \int_{A_{\mathrm{eps\,max}}}^{\infty} p_{A|M_i,T_j,k}(u)\,du \qquad \text{Eq. (2-78)}$$

where $p_{A|Mi,Tj,k}(\cdot)$ is the probability density function of A given magnitude Mi at source k, and:

$$A_{eps\max} = \max(A_{eps}, a) \qquad \text{Eq. (2-79)}$$

R-CRISIS allows performing the epsilon disaggregation with two different approaches:

1. With an accumulated epsilon where the user defines the value of ε_1 and the procedure is done between -∞ and ε_1.
2. Between two predefined epsilon values, where the user defines the values for ε_0 and ε_1. $\varepsilon_0<\varepsilon_1$.

2.10.3 Interpretation of *ε* for other probability distributions

Usually, intensity *A* is assigned a lognormal probability distribution, so equation 2-75 can be used to compute the lower integration limit, A_{eps}. However, it admits the possibility of using four different types of probability distributions, being them: Lognormal, Gamma, Normal and Beta. In the three last cases, the meaning of ε is not unambiguously defined. In R-CRISIS, the following interpretations of ε are adopted:

<u>For the Gamma distribution</u>

$$A_{eps} = \mathrm{E}(A \mid M_i, T_j, k) + \varepsilon\sigma(A \mid M_i, T_j, k), L \geq 0 \quad \text{Eq. (2-80)}$$

<u>For the Normal distribution</u>

$$A_{eps} = \mathrm{E}(A \mid M_i, T_j, k) + \varepsilon\sigma(A \mid M_i, T_j, k) \quad \text{Eq. (2-81)}$$

<u>For the Beta distribution</u>

$$A_{eps} = \mathrm{E}(A \mid M_i, T_j, k) + \varepsilon\sigma(A \mid M_i, T_j, k), 0 \leq L \leq 1 \quad \text{Eq. (2-82)}$$

In the three cases, E(*A*|*Mi, Tj, k*) and σ(*A*|*Mi, Tj, k*) are, the expected value and the standard deviation of *A* given magnitude *Mi* at source *k*, respectively.

2.11 Cumulative Absolute Velocity filter

It is common practice in PSHA to define a threshold magnitude, M_O, to determine from what magnitude on, earthquakes can produce damages in the structures and components of a dwelling in order to only consider those while performing the hazard analyses. Nevertheless, EPRI (2006) proposed that as an alternative to using M_O the Cumulative Absolute Velocity (CAV) can be used. Its value is given by the integral of the absolute value of a strong ground motion recording. There is some agreement that damaging events are those with CAV>0.16 g-sec and for that, the CAV filtering method states that the exceeding probabilities of given values of intensity, *a*, should be filtered by the probability that CAV>C_O given that a ground motion, with that level of intensity, has occurred. That probability is computed by means of a special type of attenuation relationship that relates the CAV with magnitude, *M,* and distance, *R,* (IRSN, 2005; Kostov, 2005).

For a single source, when the hazard integral is formulated in terms of exceedance rates of accelerations, *a*, this minimum magnitude is included in the following way:

$$\nu(a) = \lambda_0 \int_{M_0}^{M_U} \int_{R\min}^{R\max} f_m(M) f_R(R) \Pr(A > a \mid M, R) dRdM \quad \text{Eq. (2-83)}$$

where $v(a)$ is the exceedance rate of acceleration a, $f_m(.)$ and $f_R(.)$ are the density of magnitude, M, and distance, R, respectively, and λ_0 is the exceedance rate of earthquakes with $M>M_0$ in the seismic source.

A typical value for M_0 adopted in seismic hazard studies is M_W=5.0. But, as indicated in EPRI (2006), as an alternative to using earthquake magnitude to determine non-damaging earthquakes, it is proposed to use the ground motion measure denoted as Cumulative Absolute Velocity (CAV), given by the integral of the absolute value of a ground motion acceleration recording. To make the CAV value representative of strong ground shaking rather than coda waves the definition of CAV was later restricted to computing CAV for 1-second time windows that have amplitudes of at least 0.025g.

Although the logic behind using CAV filtering is relatively complex (see EPRI, 2006), the general idea in a few words is that the only ground motions that should contribute to the hazard estimations are those with the capability of producing damage to structures; furthermore, there is some agreement in the fact that damaging motions are those with CAV>0.16 g-sec. In view of this, the CAV filtering method states that the exceeding probabilities of given values of intensity a should be weighted (filtered) by the probability that CAV>C_0 given that a ground motion with that level of intensity, a, took place.

Although there are other possible approaches, in R-CRISIS the following CAV filtering strategy is used:

$$\nu_F(a) = \lambda_0 \int_{M_0}^{M_U} \int_{R\min}^{R\max} f_m(M) f_R(R) \Pr(A > a \mid M, R) \Pr(CAV > C_0 \mid M, R) dR dM \qquad \text{Eq. (2-84)}$$

where $\nu_F(a)$ is the filtered exceedance rate and $\Pr(CAV>C_0|M,R)$ is the probability that CAV is greater than the threshold value (taken as 0.16g-sec) given that an earthquake with magnitude M took place at distance R. In other words, the probability of having a damaging ground motion given that an earthquake of these characteristics took place.

This probability is computed by means of a special kind of attenuation relations that relate CAV with M and R. This is the case, for example, of the equation defined by the IRSN (2005) using the seismic data of the RFS 2001-01. It is also the case of the equation proposed by Kostov (2006), using the European ground motion database (Ambraseys et al., 2004).

Currently, R-CRISIS uses the following two filtering formulas:

<u>For surface-wave magnitude, M_S</u>

$$\Pr(CAV > C_0 \mid M, R) = \begin{cases} 1 & \text{if } M \le 5.5 \\ 1 - \Phi(z) & \text{if } M > 5.5 \end{cases} \qquad \text{Eq. (2-85)}$$

where:

$$z = \frac{Log(C_0) - Log(C \mid M, R)}{\sigma}$$ Eq. (2-86)

$$Log(C \mid M, R) = 0.4354M + 0.0018R - Log(R) - 0.901$$ Eq. (2-87)

where C0=1.6 m/s and σ=0.302

In the above formulas, $M=M_s$ and R is the focal distance, while $F(.)$ is the standard Gaussian probability distribution. This equation was fitted using the RFS-2001.01 (Berge-Thierry et al., 2004) database.

For moment magnitude, M_W

Make (as proposed by Scordilis (2006)):

$$M = \frac{M_W - 2.07}{0.67}$$ Eq. (2-88)

And use the above-mentioned formulas.

2.12 Logic trees

In the context of R-CRISIS, each branch of a logic tree is formed by one data file together with a measure of the degree of belief that the user has on each of the branches of being the "true" one. Results from the different branches, along with the weights assigned to each branch, are computed using the combination rule described next.

Assume that the probability of exceeding level a of intensity measure A at a computation site, in the i^{th} time frame, according to the j^{th} branch of a logic tree is $P_{ij}(A>a)$. Assume also that the probability of being the true one assigned to the j^{th} branch is w_j, j=1,...,N.

Then, the expected value of $P_{ij}(A>a)$ once all branches have been accounted for, $P_i(A>a)$, is given by:

$$P_i(A > a) = \sum_{j=1}^{N} P_{ij}(A > a) \cdot w_j$$ Eq. (2-89)

Results of the logic-tree combination will be given in the form of a new hazard model, with an associated *.dat file that will have the base name of the logic-tree file that described the combination but with the extension *.dat.

Note: it is required that the N weights add up to 1.0.

This resulting hazard model can be loaded into R-CRISIS and the corresponding hazard results can be analyzed with it (in order to obtain hazard maps, exceedance probability curves, uniform hazard spectra) as if they were the results of a regular *.dat file. Disaggregation results, however, cannot be obtained for the hazard resulting from the logic-tree combination.

Note: for a better understanding of the underlying framework of logic trees in R-CRISIS, a careful reading of the paper published by Bommer et al. (2005) is suggested.

2.13 Optimum spectra

Although establishing the design coefficient values associated to a fixed return period by means of probabilistic methodologies is a remarkable step towards the achievement of seismic safety, they do not necessarily lead to optimum design coefficients, which, as proposed by Esteva (1970) are optimal if they minimize the sum of the expected cost associated to the decision of having used that value in the design of the structure. This said in other words, means that an optimum design is that one which minimizes the sum of the initial construction cost and the net present value of the future losses because of earthquakes.

Following the methodology proposed by Rosenblueth (1976) and Whitman and Cornell (1976), to estimate the optimum earthquake design coefficients, a PSHA is first needed to be performed in R-CRISIS to obtain the hazard intensity rates *ν(a)* at the locations where the design coefficients are to be established. Then, after establishing a set of descriptors that account for the cost of the structures as a function of the design coefficient and by selecting an appropriate discount rate to consider the value of money in the future, it is possible to obtain optimum values for those design coefficients.

The methodology implemented in R-CRISIS follows the next assumptions:

1. The earthquake occurrence in the future is characterized by means of a Poissonian process
2. The initial cost of the building as well as the cost of future losses because of earthquakes depend only on one parameter, c, which is the nominal design resistance quantified in terms of the base shear
3. Time starts for every building once its construction phase has finished, and,
4. Every time the seismic demand exceeds the capacity, there is a total loss of the building.

The optimum design approach explicitly accounts for the economic factors involved during the construction and life-service time of a building; this is done by selecting the coefficient value that minimizes the initial construction cost, C_I as well as the one associated to the future losses because of earthquakes, C_{FL}. The total cost of the structure C_T is thus the sum of both.

$$C_T = C_I + C_{FL} \qquad \text{Eq. (2-90)}$$

Since all the costs are function of the design coefficient, *c*, they are denoted as $C_I(c)$, $C_{FL}(c)$ and $C_T(c)$ and then, equation 2-81 can be rewritten as

$$C_T(c) = C_I(c) + C_{FL}(c)$$ Eq. (2-91)

Figure 2-34 explains schematically the optimum coefficient approach where the red line, representing C_I, increases as *c* does whereas the blue line, representing C_{FL}, decreases as *c* increases. Finally, the green plot represents the utility function to be optimized and from where the optimum value of *c* is obtained.

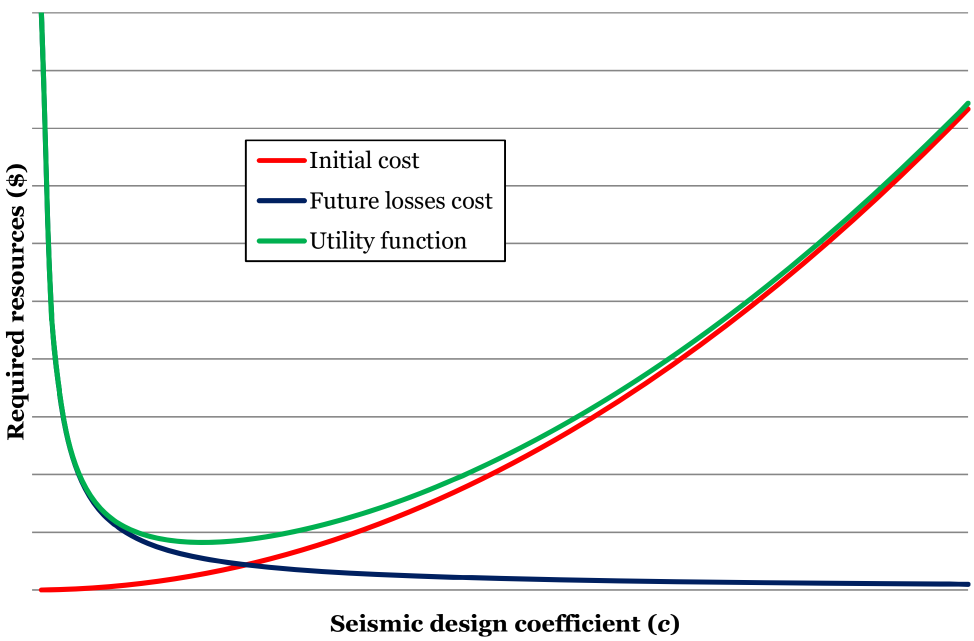

Figure 2-34 Optimum design framework

If the building was to be designed only by considering the gravitational loads, there would still be a cost associated to it, here forth referred to as C_o. That same building will also have an implicit lateral resistance, which under this framework is considered as free of charge and denoted as c_o. The initial cost of the structure can be then calculated as

$$C_I(c) = C_o + C_{Res}(c - c_o)^{\alpha}$$ Eq. (2-92)

where C_{Res} is the cost of the planned and paid lateral resistance and α is a parameter that considers the cost increase of the structure with increasing design coefficient. If equation 2-92 is normalized by C_o, it can be rewritten as

$$\frac{C_I(c)}{C_o} = 1 + \frac{C_{Res}}{C_o}(c - c_o)^{\alpha}$$ Eq. (2-93)

and, if the ratio between C_{Res} and C_o is denoted as ε, equation 2-81 finally transforms into

$$\frac{C_I(c)}{C_o}=1+\varepsilon(c-c_o)^{\alpha}$$ Eq. (2-94)

Within this methodology, it is assumed that $c \geq c_o$ since the latter is generally very low.

The net present value of the future losses of the building because of earthquakes needs to be calculated and it is also a function of the design coefficient. $NPV_{FL}(c)$ is then calculated as

$$NPV_{FL}(c)=C_I(c)\cdot(1+S_L)\cdot\frac{\nu(c)}{\mu}$$ Eq. (2-95)

where S_L accounts for secondary losses and those that occur due to human losses, $\nu(c)$ is the exceedance rate of the seismic demand and μ is the discount rate that considers the value of money in the future.

Once the optimum value of *c* has been established, its associated mean return period is obtained from the hazard plot at each location. This leads to seismic hazard maps which values have variable mean return periods that are reflected in a smoother transition between adjacent zones.

Finally, the mean return period variable is truncated to a minimum and maximum value, T_{Min} and T_{Max}. The first one to follow the building code philosophy of establishing minimum requirements while the second one is used to avoid the appearance of accelerations associated to not feasible earthquakes in zones of very low seismic activity.

2.14 Stochastic catalogue generator

Based on the geometry and seismicity parameters assigned to each of the sources, and when Poissonian occurrence models have been assigned to them, it is possible in R-CRISIS to generate stochastic catalogues. These catalogues represent a possible realization of a random occurrence in space and time within a defined duration (in years) specified by the user.

The generation of the stochastic catalogues is available when using any Poissonian seismicity model in combination with any of the following geometric models:

- Line fault
- Rectangular fault
- Area-planes
- Point
- Area
- Slabs
- Grids

One relevant aspect when generating stochastic catalogues is guaranteeing that the events are compatible with the base information in the sense that, for instance, those events occur only within the boundaries of the seismic sources and that the magnitudes and number of events in each observation timeframe, are in line with the recurrence models that were used to characterize the earthquake occurrence at each source. Next, a description of the validation processes for the location, magnitude and number of events followed when implementing this feature in R-CRISIS is presented.

2.14.1 Validation of location of events

The validation of the location of generated events using this feature in R-CRISIS was validated for all the possible geometry models. In all cases a duration of 100 years was used and different shapes, including complex geometries, were used. First, Figure 2-35 shows the validation for the case of a line-fault where the geometry of the source is displayed as the red line whereas the epicenters correspond to the blue dots.

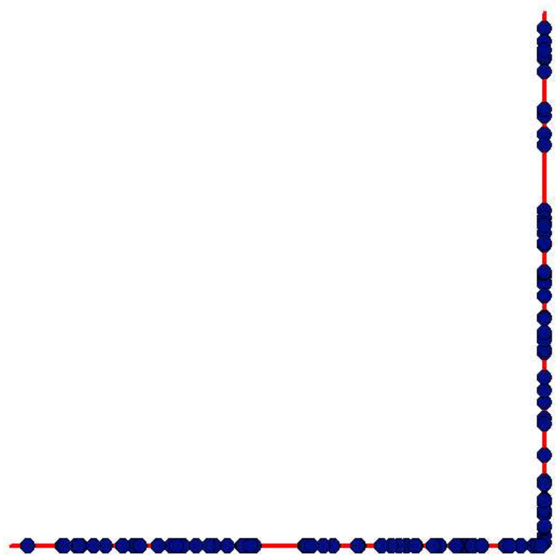

Figure 2-35 Validation of the location for the stochastic catalogue generated for line faults.

Figure 2-36 shows the validation for the case of a rectangular fault, with the upper lip as indicated in the red line, with dip of 45° and width of 20km; the epicenters in this case correspond again to the blue dots. The depth of the events varies in accordance to the inclined plane formed by this rectangular fault.

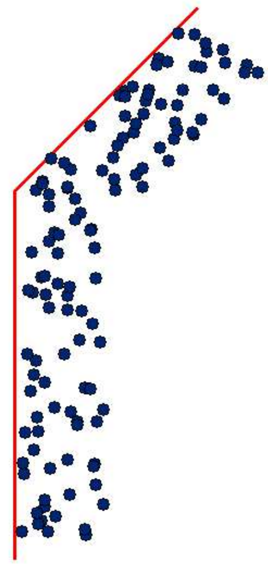

Figure 2-36 Validation of the location for the stochastic catalogue generated in a rectangular fault

Figure 2-37 shows the validation for the case of an area-plane with complex geometry. The boundaries of the source are depicted by the red polygon whereas the epicenters correspond to the blue dots.

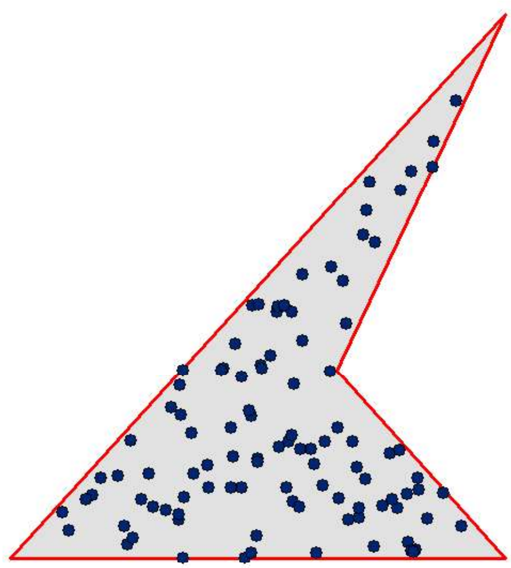

Figure 2-37 Validation of the location for the stochastic catalogue generated in an area-plane

Figure 2-38 shows the validation for the case of point sources (SSG) where the location of the sources is depicted by the red squares whereas the epicenters associated to the stochastic catalogue by the blue dots.

Figure 2-38 Validation of the location for the stochastic catalogue generated in point sources (SSG)

Figure 2-39 (left) shows the validation for the case of area sources where behavior is set as normal (ruptures can go beyond the boundaries of the source). The boundaries of the source are depicted by the red polygon whereas the epicenters by the orange dots. Figure 2-39 (right) shows the validation for the case of again, area sources, but now with the behavior set as treat as fault. In the second case, it is evident that epicenters (depicted by blue dots) are not that close to the boundaries of the polygon if compared to the normal behavior case.

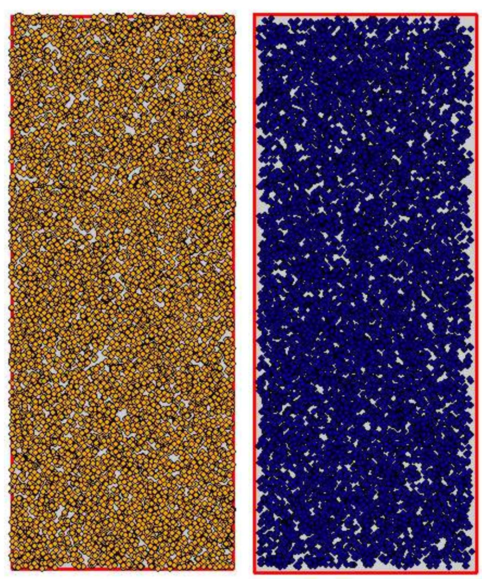

Figure 2-39 Validation of the location for the stochastic catalogue generated in area sources. Left: normal behavior. Right: treat as fault behavior

Figure 2-40 shows the validation for the case of a slab source comprised by three slices which dip is equal to 80° and have all an equal width of 15km. The upper part of the slab is depicted by the red polygon whereas the blue dots correspond to the epicenters. From the latter it is possible to visualize the geometry and alignment of the three slices that are part of this source.

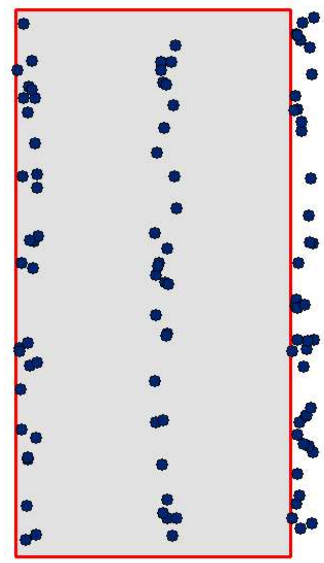

Figure 2-40 Validation of the location for the stochastic catalogue generated in slab sources

Figure 2-41 shows the validation for the case of a grid source which boundaries are depicted by the red polygon. Epicenters (shown as blue dots) occur only at the location of the nodes of the grid, in this case with equal spacing in both orthogonal directions. Depths are the same (as of the grid) for all the events.

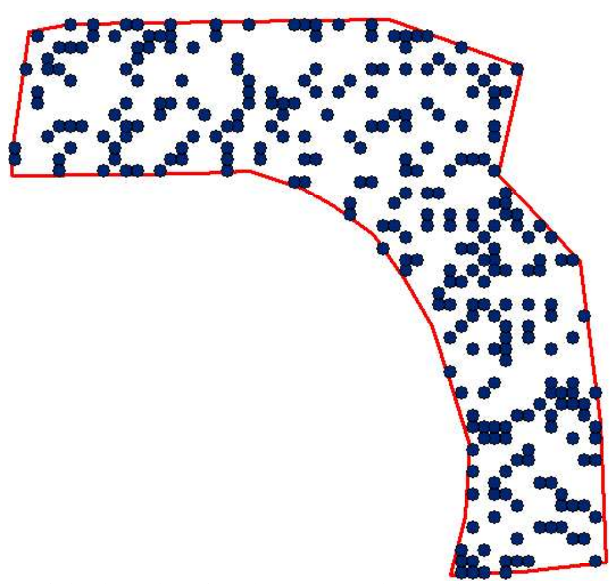

Figure 2-41 Validation of the location for the stochastic catalogue generated in gridded sources

2.14.2 Validation of magnitude and number of events

Figure 2-42 shows the comparison of the modified G-R recurrence relationships for a source which seismic parameters λ_0, β and M_U are 1.0, 2.0 and 8.0 respectively, and those estimated using the maximum likelihood methodology (McGuire, 2004) for a stochastic catalogue of 100 years duration. Knowing that 100 years is not a long enough observation window, it should not be a surprise that moderate to large earthquakes, although feasible of occurring at that source, are not part of the events included in the stochastic catalogue. λ_0 and β for the stochastic catalogue with 100 years duration are in this case equal to 1.02 and 2.17, respectively.

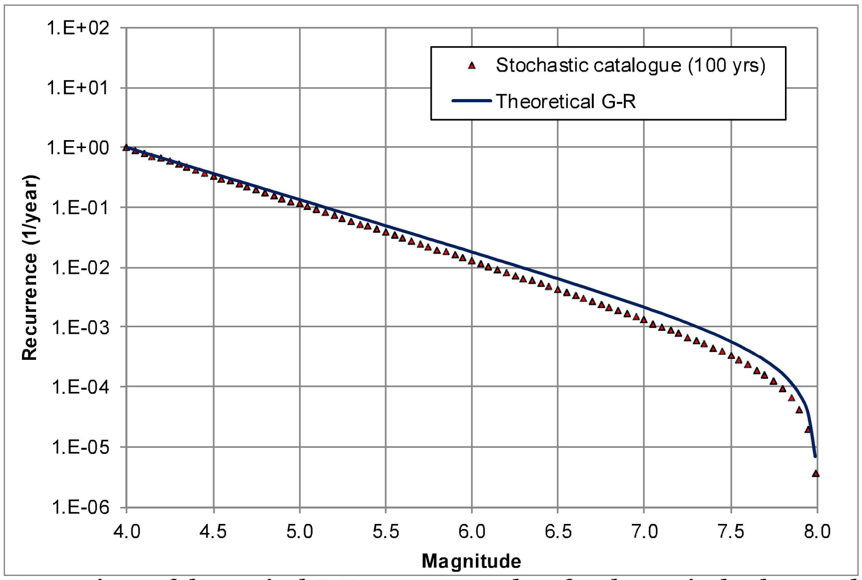

Figure 2-42 Comparison of theoretical G-R recurrence plots for theoretical values and a stochastic catalogue with 100 years duration

If the duration of the catalogue is increased to a long enough timeframe (e.g. 10000 years), the same comparison yields the results shown in Figure 2-43, matching almost exactly the theoretical values. λ_0 and β for the stochastic catalogue with 10000 years duration are in this case equal to 1.01 and 2.02, respectively.

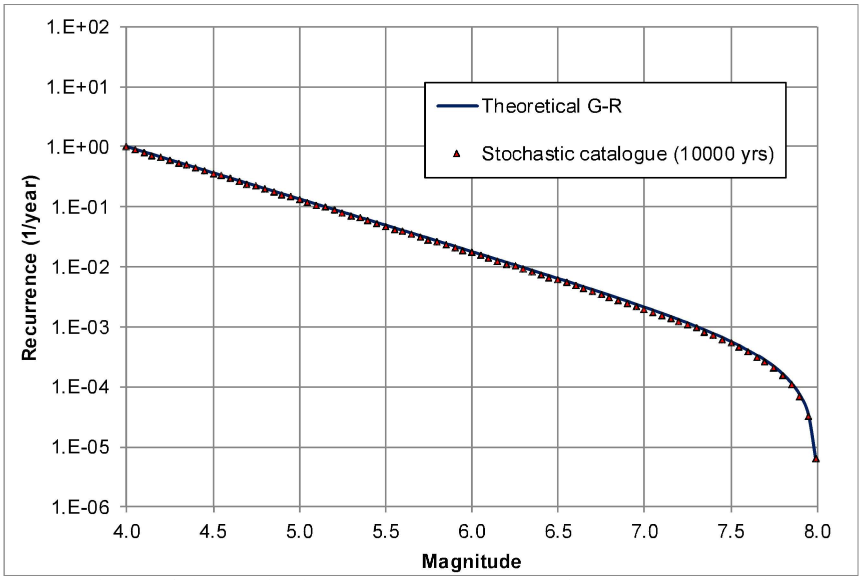

Figure 2-43 Comparison of theoretical G-R recurrence plots for theoretical values and a stochastic catalogue with 10000 years duration

2.15Conditional mean spectrum

The conditional mean spectrum (CMS) is a spectrum that incorporates correlations across periods to estimate the expected pseudo acceleration values, *Sa*, at all periods *T*, given the target Sa value at the period of intesterest *T**, *Sa*(*T**). R-CRISIS implements a procedure to calculate the "exact" conditional spectrum (CS) instead of the CMS, which uses mean values of M, R and other parameters related to the GMPEs.

R-CRISIS calculates the exact CS following the aggregation approach method proposed by Lin et al. (2013), which uses the same event set used in the PSHA computation to aggregate the hazard. To calculate the CS, it is necessary to:

- Define the calculation site. R-CRISIS will set the calculation site as the city or grid point that lies closest to the click point.
- Define the period of interest: choose the period of interest, T*, for which the CS will be calculated. The periods for which the CS calculation are available are those defined for the PSHA in R-CRISIS.
- Set either the target intensity, Sa(T*), or the exceedance probability, Pe. Choose the intensity value for which CS results will be presented or choose the desired exceedance probability (R-CRISIS will compute the exceedance probability if the intensity is given, or the intensity if the exceedance probability is provided).
- Choose the inter-period correlation model: in the calculation model, it is necessary to establish the inter-period correlation model ρ(T*,T). Two models are available for this in R-CRISIS, the one by Baker and Jayaram (2008) and the model by Jaimes and Candia (2019).

Once these parameters are defined, R-CRISIS will calculate the CS, given a target value at the period of interest, Sa(T*), using the following equation.

$$\mu \ln Sa(T_i) \,|\, \ln Sa(T^*) = \sum_k \sum_j p^d_{j,k} \times \mu \ln Sa_{j,k}(T_i) \,|\, \ln Sa(T^*) \qquad \text{Eq. (2-96)}$$

where $p^d_{j,k}$ is the mean annual exceedance frequency of the j[th] event (earthquake) and k[th] logic-tree branch, normalized by the total aggregated hazard and,

$$\mu \ln Sa(T_i) \,|\, \ln Sa(T^*) = \mu \ln Sa_k(M_j, R_j, \theta_j, T_i) + \rho(T^*, T_i) \cdot \varepsilon_j(T^*) \cdot \sigma \ln Sa_k(M_j, \theta_j, T_i)$$

Eq. (2-97)

where $\mu_{\ln Sak}$ is the natural logarithm of the intensity *Sa* associated to event *j* given a magnitude M_j, distance R_j, other parameters θj and spectral period T_i. $\rho(T^*,T_i)$ is the correlation between the period of interest, T^*, and the spectral period T_i, $\varepsilon_j(T^*)$ is the number of standard deviations b which $\ln Sa(T_i)$ differs from the mean spectral ordinate predicted by a given GMPE, $\mu_{\ln Sa}(M,R,\theta,T_i)$, at T_i.

$$\varepsilon(T_i) = \frac{\ln Sa(T_i) - \mu_{\ln Sa}(M, R, \theta, T_i)}{\sigma_{\ln Sa}(M, \theta, T_i)} \qquad \text{Eq. (2-98)}$$

$\sigma_{\ln Sak}(M_j,\theta_j,T_i)$ is the standard deviation of $\mu_{\ln Sak}(M_j,R_j,\theta_j,T_i)$. Finally, the standard deviation associated to the CS is also calculated as:

$$\sigma_{\ln Sa} \,|\, \ln Sa(T^*) = \sqrt{\sum_k \sum_j p^d_{j,k} \left[\sigma^2_{\ln Sa\, j,k(Ti)|\ln Sa(T^*)} + (\mu_{\ln Saj,k(Ti)|\ln Sa(T^*)} - \mu_{\ln Sa(Ti)|\ln Sa(T^*)})^2 \right]}$$

Eq. (2-99)

where,

$$\sigma_{\ln Saj,k(Ti)|\ln Sa(T^*)} = \sigma_{\ln Sak}(M_j, \theta_j, T_i) \times \sqrt{1 - \rho^2(T_i, T^*)} \qquad \text{Eq. (2-100)}$$

As the reader might have noted, all the calculation process has been done in terms of the natural logarithm; this happens because it is assumed that the GMPEs involved follow a lognormal probability distribution. Therefore, when using GMPEs that are not lognormally distributed (e.g., truncated, gamma, hybrid, etc.), the CS will only be computed approximately.

Figure 2-44 shows an example of CS calculation for 2.0s spectral period, 143 cm/s2 target intensity and Jaimes and Candia (2019) inter-period correlation model. Curves of CS ± one standard deviation are also plotted.

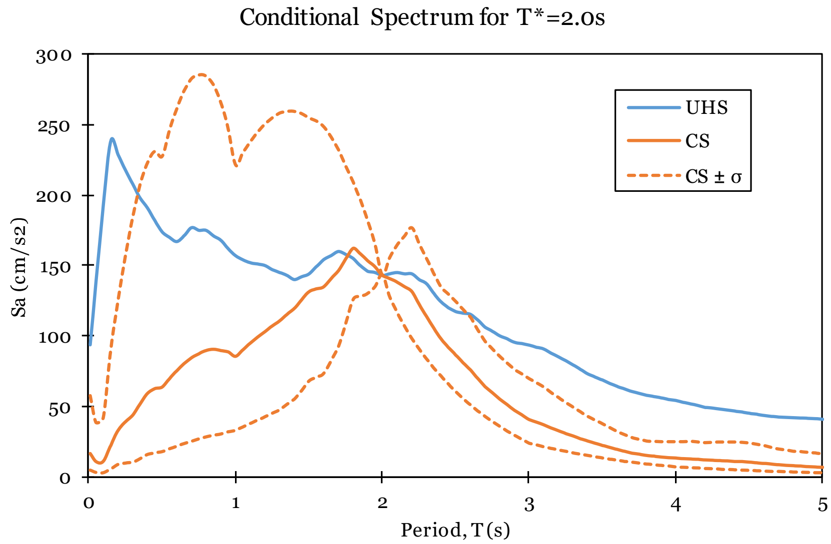

Figure 2-44 Conditional Spectrum for T*=2.0s and Jaimes and Candia (2019) inter-period correlation model

2.16 Probabilistic liquefaction hazard analysis

Most of the methods commonly used to assess the susceptibility to liquefaction aim to estimate the safety factor (*Fs*) against liquefaction, or the probability of liquefaction occurrence triggered by an earthquake with known parameters, once the relevant characteristics of a soil profile are available. This approach usually considers only one event, usually referred to as the Maximum Credible Earthquake (MCE) and therefore, it is impossible to know how frequently liquefaction can occur since there is just a vague link between the MCE and its frequency of occurrence.

The safety factor against liquefaction, *Fs*, is estimated as:

$$F_S = \frac{CRR}{CSR} \qquad \text{Eq. (2-101)}$$

where *CRR* is the Cyclic Resistance Ratio and *CSR* the Cyclic Stress Ratio.

Since there are many other earthquakes besides the MCE that can contribute, with a non-negligible share, to the liquefaction probability, R-CRISIS allows a rigorous probabilistic liquefaction hazard analysis (PLHA) that is performed within a framework mostly taken from PSHA and using an event-based approach. On this approach, the effects of multiple (generally thousands of) earthquakes with different magnitudes, locations and occurrence frequencies are considered, knowing also that the ground motions of these earthquakes can be only predicted with large uncertainties and that site effects can modify seismic waves.

Several authors have proposed ways to adapt the deterministic models to probabilistic frameworks and proposed empirical expressions to estimate the liquefaction probability for a given event. For instance, Ku et al. (2012) have proposed the following expression, which is the probabilistic version of the Robertson and Wride method for liquefaction evaluation:

$$P_L = \frac{1}{1+\left(\frac{Fs}{0.9}\right)^{6.3}}$$ Eq. (2-102)

Where P_L is the probability of experiencing liquefaction given that the earthquake characterized by the a_{max} and M values has occurred.

In this methodology, the same framework of PSHA is followed but for a better understanding, the hazard analysis is performed by summing individual events rather than in terms of integrals. Then, the annual frequency of occurrence of liquefaction, at a given depth, z, $\nu_L(z)$, can be estimated as:

$$\nu_L(z) = \sum_{i=i}^{N} \Pr(Liquefaction\ at\ depth\ z \mid Event\ i) \cdot F_{ai}$$ Eq. (2-103)

where N is the total number of events that are part of the stochastic catalogue, Pr(*Liquefaction at depth z*|*Event$_i$*) is the probability of experiencing liquefaction at depth z given that the i^{th} event occurred and F_{ai} is the annual occurrence frequency of the i^{th} event.

An individual earthquake is characterized by several parameters, θ, such as its magnitude, hypocentral location, rupture area and orientation of the fault plane, among others. Therefore, the term Pr(*Liquefaction at depth z*|*Event$_i$*) requires calculating the liquefaction probability for an event with given θ parameters and not only by an event defined by its a_{max} and M values. Within a PSHA framework, a_{max} is usually modelled as a random variable to account for uncertainties in the GMPM, in view of which, Pr(*Liquefaction at depth z*|*Event$_i$*) is computed as:

$$\Pr(Liquefaction\ at\ depth\ z \mid Event\ i) = \int_0^{\infty} \Pr(Liquefaction\ at\ depth\ z \mid a_{\max}, M) \cdot p(a_{\max} \mid \theta_i)\, da_{\max}$$ Eq. (2-104)

where p(a_{max}|θ_i) is the probability density function of a_{max} given the parameters θ_i that characterize this event. This PDF is usually furnished by the GMPM (or combination of GMPMs) that is being used and, very importantly, by a soil response analysis since a_{max} is the PGA at the surface of the soil deposit whose liquefaction potential is being assessed. On the other hand, Eq. 2-102, 2-103 and 2-104 illustrate the linkage between conventional liquefaction analysis methods and PSHA. These equations are useful to estimate the annual occurrence frequency of liquefaction, not triggered by a single event but in a complex seismic environment characterized by a stochastic earthquake catalogue and one or more GMPMs.

In the current version, R-CRISIS has implemented the liquefaction probability estimation after Robertson and Wride, but any other approach that allows estimating liquefaction probabilities can be implemented within the above explained framework.

Typical results of a PLHA are shown in Figure 2-45. where, for different depths, the annual occurrence rate of liquefaction, the return period of the liquefaction occurrence and the probability of liquefaction occurring within a timeframe of 50 years are shown.

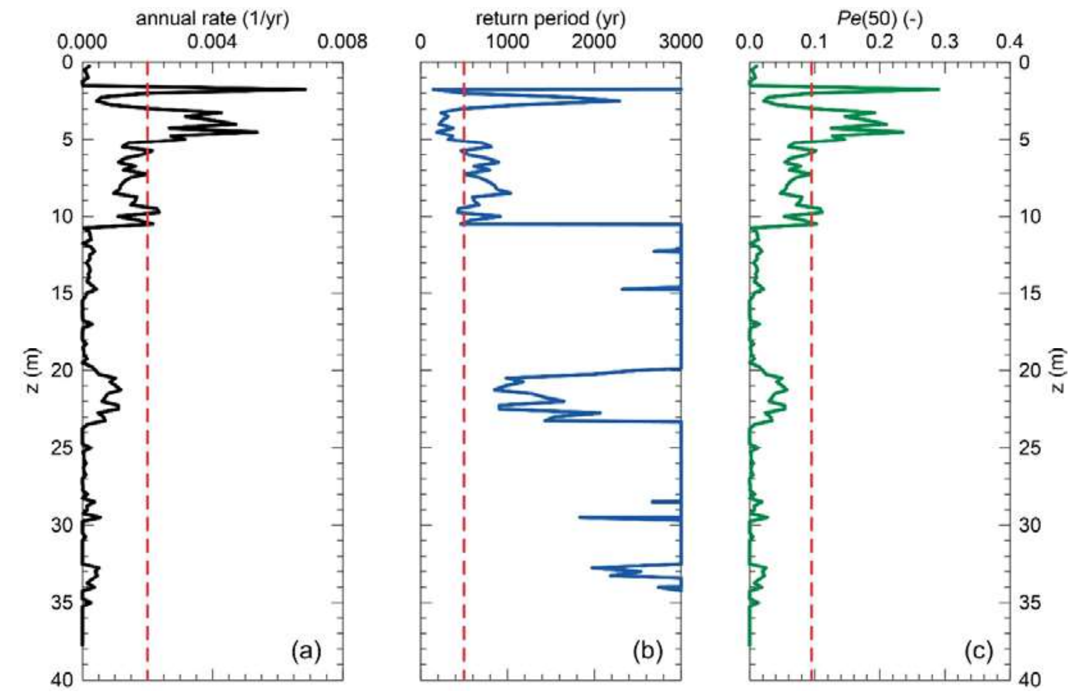

Figure 2-45 PLHA results in terms of annual exceedance rates (a), return periods (b) and occurrence probability in the next 50 years (c)